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TECHNICAL REPORT

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DEVELOPMENT AND ANALYSIS OF TANK EVASION STRATEGIES IN MISSILE EFFECTIVENESS MODELS

by M. Falco and G. Carpenter

Research Department
Grumman Aerospace Corporation
Bethpage, New York 11714

Contract DAAE07-76-C-0106

U.S. ARMY TANK-AUTOMOTIVE
RESEARCH AND DEVELOPMENT COMMAND
Warren, Michigan 48090

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June 1977

Approved by: Richard A. Scheuing

Richard A. Scheuing
Director of Research

ABSTRACT

This report presents a new methodology with which to analyze the survivability of tank vehicles to anti-tank missile threats. The approach employs elements of optimal control theory, stochastic learning theory, and dynamical simulation in a computational method which determines tank evasive maneuvering strategy as an integral part of the survivability analysis.

The method develops an optimal strategy in the sense of maximizing tank survival probability for all missile launch conditions. The strategy is in the form of a feedback control policy based upon a discretized set of information states which are assumed available to the tank commander as visual or warning system cues.

Computational results for both the survivability and associated optimal evasive maneuvering are presented for an M-60 class tank vehicle and an anti-tank missile representative of an upgraded foreign threat system. The results illustrate how the methodology can be employed to assist in quantifying survivability tradeoffs involving tank threat warning systems, evasive maneuver computer systems, and acceleration, deceleration, and turning performance specifications.

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LIST OF SYMBOLS

a = tank acceleration control variable defined in Table 1

a^+ = tank maximum linear acceleration

a^- = tank maximum braking deceleration

A_1, A_2, B = tank envelope length and width parameters

g = gravitational acceleration

K = rate compensator gain

K_1 = tank maximum pivot turn rate ($V_T = 0$)

K_2 = tank maximum lateral acceleration in turning ($V_T > 0$)

K_{ap} = missile autopilot/airframe gain

l = missile guidance wire payout length

l_{MAX} = missile guidance wire maximum length

n_{LIM} = missile lateral maneuver limit (g's)

p_{ij} = probability of selecting control u_j whenever the tank is in the state x_i

p_K = overall kill probability of missile warhead

r = missile airframe yaw rate

R = missile to tank line of sight (L.O.S.) range

R_M = gunner to missile L.O.S. range

R_{MIN} = missile/tank trajectory minimum miss distance (from respective e.g.)

R_p = gunner/missile L.O.S. vector projection on gunner/tank L.O.S.

R_T = gunner to tank L.O.S. range

t = missile elapsed flight time

t_{MAX} = missile maximum guidance time

u_j = generic elemental control in the tank decision table

V_M = missile velocity

$V_{M\hat{x}}$ = missile velocity component along the $\hat{x}$ axis

$V_{M\hat{y}}$ = missile velocity component along the $\hat{y}$ axis

V_N = tank velocity component magnitude normal to gunner/tank L.O.S. vector

V_R = missile/tank relative velocity

$V_{R\hat{x}}$ = missile/tank relative velocity component along the $\hat{x}$ axis

$V_{R\hat{y}}$ = missile/tank relative velocity component along the $\hat{y}$ axis

V_T = tank velocity

$\hat{x}, \hat{y}$ = tank centered coordinate axes

x_i = generic state in the tank decision table

x_M, y_M = missile position components in the x, y inertial frame

LIST OF SYMBOLS (continued)

$\hat{x}_R, \hat{y}_R$ = missile position relative to the $\hat{x}$, $\hat{y}$ coordinate axes

x_T, y_T = tank position components in the x, y inertial frame

α = missile forward acceleration parameter

β = tank turning control variable defined in Table 1

γ = missile inertial heading (relative to x inertial axis)

δ = missile boresight error from target L.O.S. at launch

Δy = missile position error normal to gunner/tank L.O.S.

Δy^* = missile guidance error with lead/lag compensator

$\Delta \dot{y}^{**}$ = missile guidance error with rate and lead/lag compensation

θ = missile to tank L.O.S. inertial orientation

ν = warhead/envelope angle of obliquity measured from the surface normal at the contact point

τ_1, τ_2 = lead/lag compensator time constants

τ_{ap} = missile autopilot/airframe time constant

ϕ = tank inertial heading

Ψ_M = gunner to missile L.O.S. inertial orientation

Ψ_T = gunner to tank L.O.S. inertial orientation

ω = relative bearing of missile from tank y axis

• = dot over symbol signifies differentiation with respect to time

Note: Vector notation (e.g. $\overline{V}_T$) has been surpressed to clarify the presentation, leaving the simple task of discrimination of whether a scalar or vector form of a symbol to the reader.

1. INTRODUCTION

The rapid development and widespread operational deployment of anti-tank guided missiles (ATGMs) has increased the need for a better computational tool with which to 1) quantify tank survivability and 2) find optimal evasive tactics against these threats. To be effective, this tool should have the ability to quantify the sensitivity of vehicle survivability to vehicle maneuvering performance and countermeasure variations for arbitrary levels of warning system sophistication. Moreover, the optimal evasive maneuvering/countermeasure strategy which maximizes survival probability should be constructed as an integral part of the survivability determination.

Grumman has adapted the methodology developed in air-to-air combat analyses (Refs. 1 and 2) for the tank vehicle-ATGM application. The methodology comprises extensive tank/missile encounter simulations together with a stochastic learning procedure for deriving the optimized evasive strategies for the tank. This methodology permits model realism not generally attainable in other optimization approaches: the ability to include threat warning information in the maneuver and countermeasure strategy development and an optimization criterion which deals directly with survival probability measures as derived from extensive missile warhead/tank envelope interaction studies. The computational approach is comprised of two phases: a learning phase and statistics phase. The learning phase constructs the tank's "optimized" evasive strategy while the statistics phase, utilizing the computed optimal strategy, quantifies the tank survivability for the entire initial condition space.

The representative tank/ATGM model considered in this report is two dimensional in nature; the missile flight path and tank trajectory is restricted to the horizontal plane. This model has also been restricted to study only tank maneuver capability; no countermeasures were allowed in the evasive strategy development. These restrictions have been arbitrarily imposed for reasons of simplification and can be removed without basic alteration of the methodology should future studies so dictate. The tank vehicle model selected for the baseline study in this report is typical of the current performance class of vehicle in operational deployment today. The baseline ATGM selected is assumed to be a man portable, optically tracked (in a semi automatic sense), command to line of sight guidance missile representative of current day foreign threats.

The computational results associated with three model variations are illustrated in this report. In the first study, called the "baseline configuration", the tank vehicle was assumed to employ relative missile range and angle off as information for the evasive strategy determination. The second study expanded the set of observables to include tank velocity information in addition to the relative range and angle off information employed in the first study. This permits a finer resolution of the tank evasive strategy in terms of the threat warning and other dynamical combat variables, and quantifies the improvement in tank survivability afforded by the expanded observables. The third study examines the tank survival sensitivity to a change in the braking deceleration capability from that employed with the baseline configuration. These results furnish a guide as to how one might proceed using this methodology to assist in the design of tank threat warning systems, a vehicle evasive maneuver computer system, and vehicle acceleration, deceleration, and turning performance requirements.

2. TANK VEHICLE MODEL

This section describes the tank vehicle coordinate nomenclature and dynamical model. In addition, the basis for the tank maneuvering strategy is developed in terms of the elemental maneuver set and observable set (threat warning threshold) definitions.

COORDINATE NOMENCLATURE AND DYNAMICAL MODEL

The coordinate geometry employed in the two dimensional horizontal plane model is shown in Fig. 1. The missile operator (gunner) is at the origin of the fixed x, y reference axes. The tank vehicle's position components, velocity, and velocity orientation for a point mass representation, are governed by the following equations

$$\dot{x}_T = V_T \sin\phi$$

$$\dot{y}_T = V_T \cos\phi$$

$$\dot{\phi} = \beta$$

$$\dot{V}_T = a$$

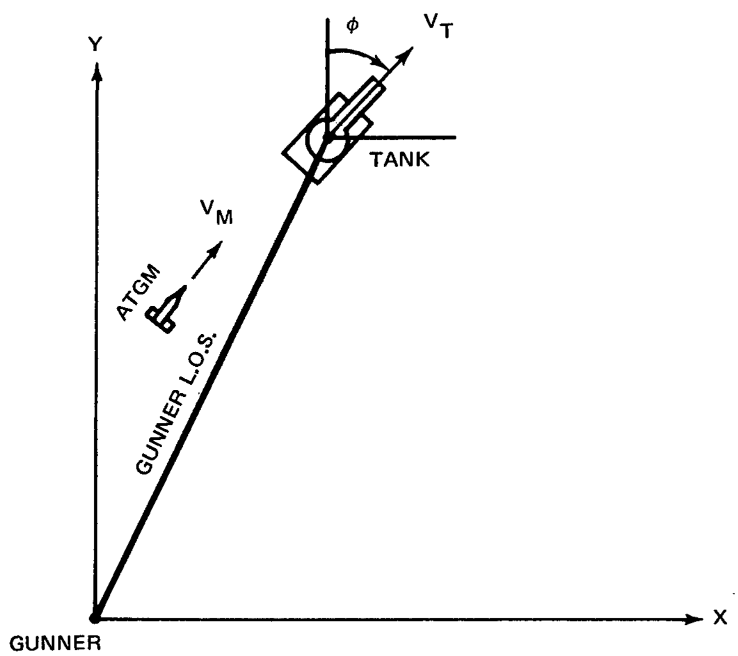


Fig. 1 Tank Coordinate Nomenclature

The quantities β and a are the turning and acceleration control variables respectively, and are fully defined in the specification of the elemental maneuvers later in section (2).

MANEUVERING STRATEGY DEVELOPMENT

The stochastic learning methodology is restricted to selection from a finite number of elemental maneuvers. It is logical to select the maximal performance maneuvers to be this finite basis set. The elemental maneuver set for the tank vehicle incorporating the maximal performance elements has been constructed as shown in Fig. 2. The particular set employed depends upon the tank velocity state at a control decision point. Two conditions for the velocity state are considered, the positive velocity case of Fig. 2a and the stationary case of Fig. 2b. In the $V_T > 0$ case, turns at maximum sustainable lateral g's at constant velocity are permitted. In addition, maximum linear acceleration (up to maximum forward velocity) and maximum deceleration to rest along the current heading are selectable. The

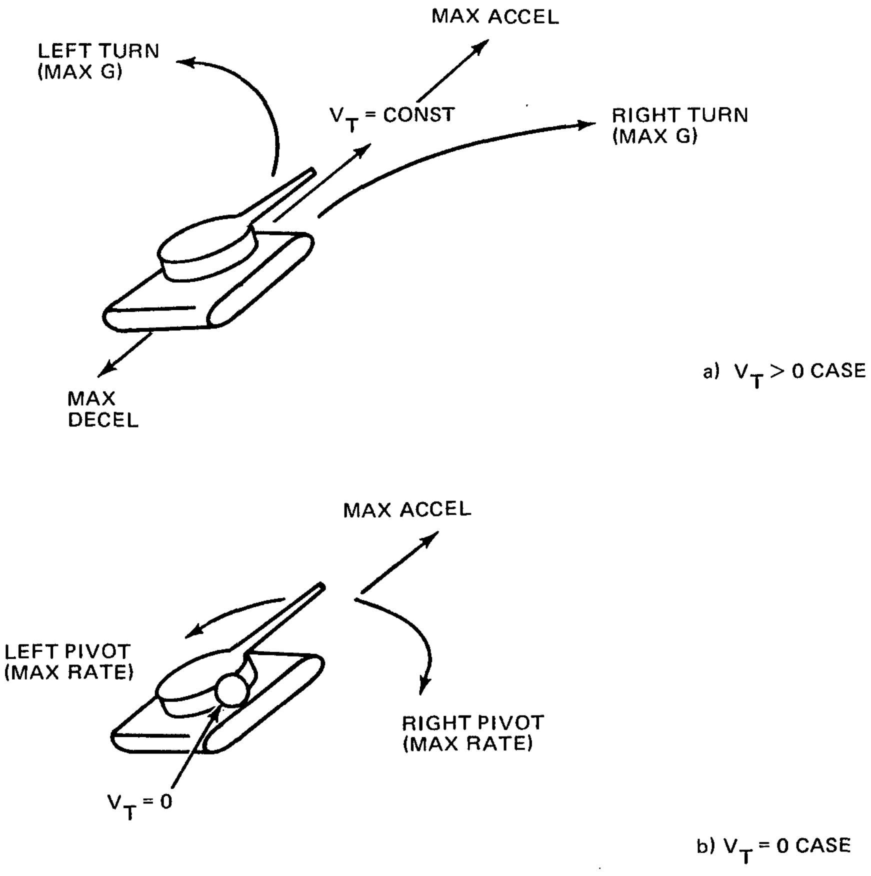


Fig. 2 Tank Vehicle Elemental Maneuvers

remaining choice is the maintenance of speed and heading at the current values. In the $V_T = 0$ case, left and right pivot turns at the maximum rotation rate are permitted. In addition, the maximal linear acceleration (to maximum forward velocity) and remain stationary choices are also included. The dynamical implementation of these basic maneuvers is accomplished via the control variables β and a. The control variable values as defined for each of the maneuver choices are given in Table 1.

TABLE 1. TANK CONTROL VARIABLE DEFINITIONS

Maneuver / Control Variable	Left Turn		Accel.		Straight		Decel.		Right Turn	
	$V_T>0$	$V_T=0$	$V_T>0$	$V_T=0$	$V_T>0$	$V_T=0$	$V_T>0$	$V_T=0$	$V_T>0$	$V_T=0$
β (turning)	$-\frac{K_2}{V_T}$	$-K_1$	0	0	0	0	0	0	$\frac{K_2}{V_T}$	K_1
a (accel. and decel.)	0	0	a^+	a^+	0	0	a^-	0	0	0

The maneuver parameters K_1, K_2, a^+, and a^- are interpreted as follows:

$K_1 \sim$ maximum pivot rate of the stationary tank in rad/sec.

$K_2 \sim$ maximum sustainable lateral turning acceleration in ft/sec^2.

$a^+ \sim$ maximum linear acceleration (velocity dependent) in ft/sec^2.

$a^- \sim$ maximum braking deceleration in ft/sec^2.

These data for the baseline configuration are given in section 5 of this report.

The maneuvering strategy development relies on the specification of which relative coordinates (tank/missile positions, rates, etc.) and corresponding threshold levels for these coordinates, will approximate the tank commander's observable information during the engagement. Once the threshold levels have been specified, a finite number of relative coordinate contingencies or "regions" are considered for the basis of the strategy development. Figure 3 gives the nomenclature associated with the relative coordinates assumed observed by the tank.

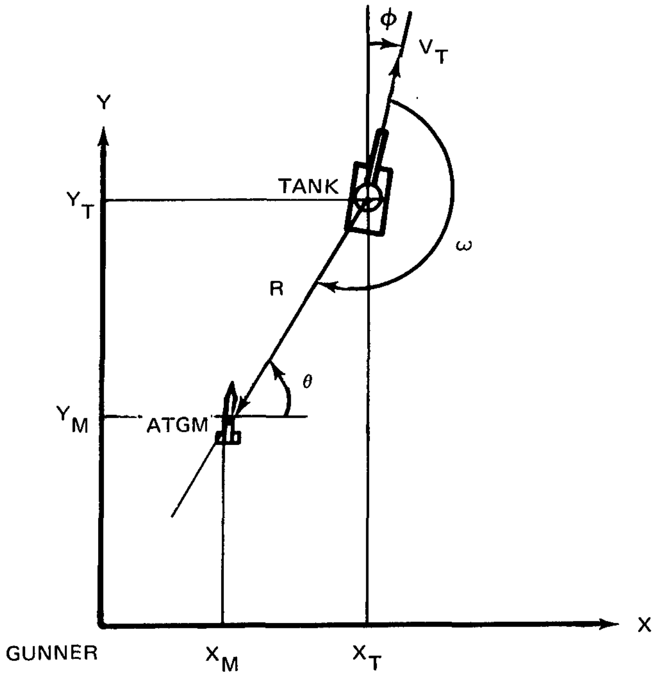


Fig. 3 Tank Observable Coordinate Nomenclature

The observable components assumed for the baseline tank are relative range (line-of-sight range) R and relative bearing ω. These quantities are computed from the following

$$R = \left[(x_T - x_M)^2 + (y_T - Y_M)^2\right]^{1/2}$$

$$\omega = \frac{3\pi}{2} - \phi - \theta$$

The baseline threshold levels for these observable coordinates are illustrated in Fig. 4. The range thresholds are in 250 m divisions out to the 2.5 km range point with an additional 500 m division to the 3.0 km range. The bearing thresholds are in 30° divisions. The baseline observable set or threat warning space therefore comprises 11 (range), 12 (bearing), or 11 x 12 = 132 contingencies or "regions". The evasive stragtegy involves the development of a decision table (Table 2) which consists of a probability distribution used in the selection of an elemental maneuver for each of the regions. When the warning system detects the missile crossing an observable threshold the tank reacts by choosing an elemental maneuver from the decision table. This maneuver is sustained until the missile enters another observable region and another maneuver selected, etc. Thus, the tank's threat warning system and its elemental maneuver performance coupled through the decision table form the model through which global evasive maneuvers are implemented.

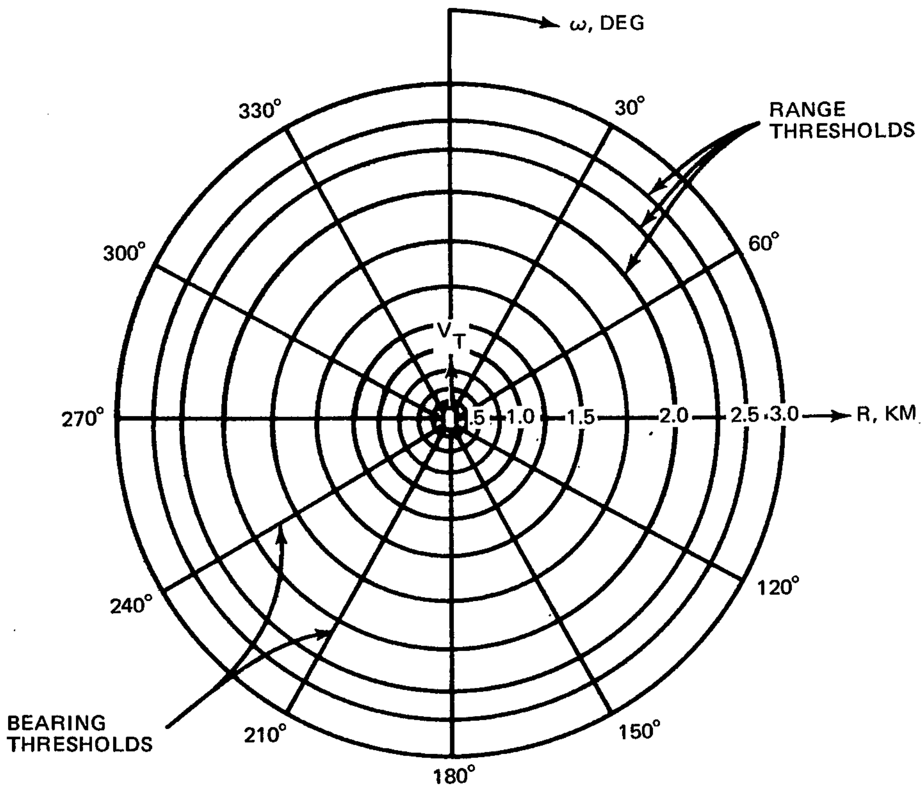


Fig. 4 Baseline Tank Observable Set

3. ANTI-TANK GUIDED MISSILE MODEL

This section describes the threat missile coordinate nomenclature and dynamical model. The missile system modeled is a postulated future threat resulting from upgrading a widely deployed man-portable system which currently employs manual target L.O.S. tracking and manual joystick control. The currently deployed system is described in Ref 3. In the upgraded system the gunner again supplies manual target L.O.S. tracking, but now the missile steering commands are automatically computed (in analog fashion) by the gunner guidance module and transmitted to the missile flight controls over the wire link. The gunner/missile system guidance parameters, aerodynamic constraints, and warhead properties as incorporated in the model are described.

COORDINATE NOMENCLATURE AND DYNAMICAL MODEL

The coordinate geometry employed in the two dimensional horizontal plane model is shown in Fig. 5. The symbol nomenclature is described in the List of Symbols (prefatory pages). The L.O.S. error and target rate quantities necessary for simulation of the command to L.O.S. guidance are defined in Fig. 6.

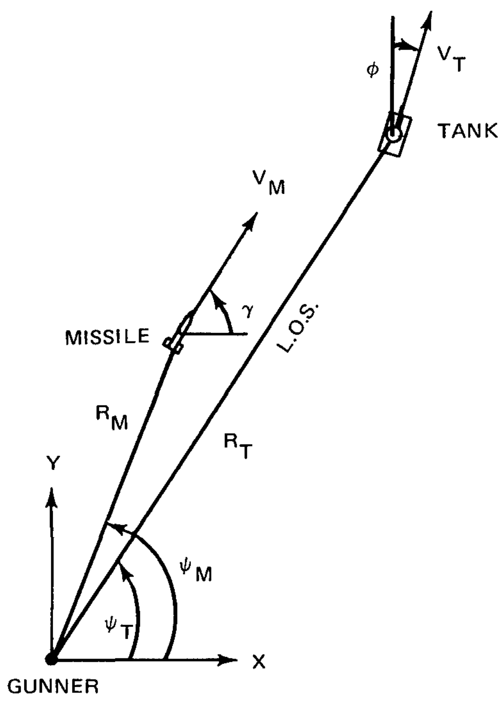


Fig. 5 Missile Coordinate Nomenclature

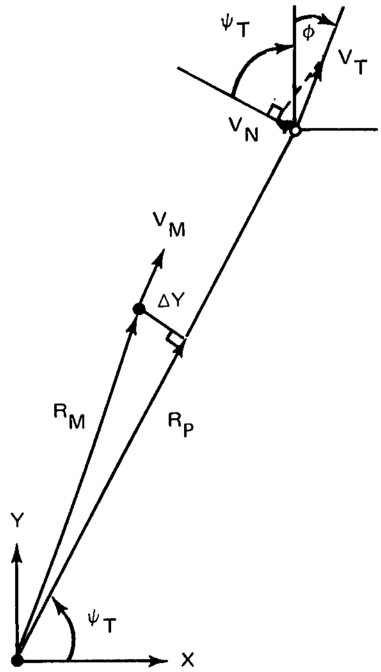


Fig. 6 Guidance Nomenclature

The two calculated errors for simulation of the guidance are missile position error, Δy, normal to the L.O.S., and target velocity normal to the line of sight, V_N. These errors correspond directly to the angular error (between missile flare and target L.O.S.) measured by the gunners' infrared optics and the angular rate measured by his optical sight rate tachometer. This development assumes perfect target L.O.S. tracking by the gunner. However, realistic gunner tracking dynamics obtained from actual missile firings against both stationary and moving targets can be incorporated in the model, if desired. The upgraded missile dynamical system as employed in the 2-D simulation is shown in block diagram form in Fig. 7. The system equations associated with the block diagram representation as employed in the digital simulation are given below.

The kinematic equations in the x, y cartesian inertial frame are

$$\dot{x}_M = V_M \cos\gamma$$
$$\dot{y}_M = V_M \sin\gamma$$
$$\dot{\gamma} = r$$
$$\dot{V}_M = \alpha$$

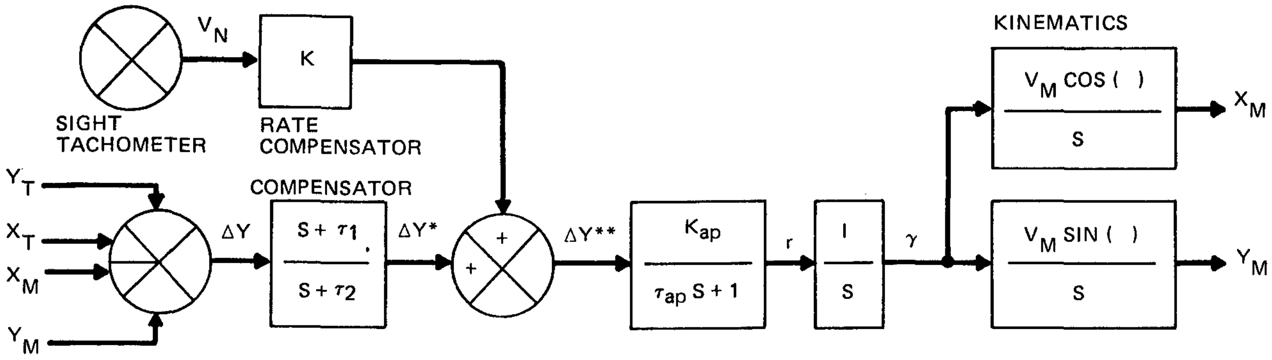


Fig. 7 Representation of Upgraded Missile System

The autopilot/airframe yaw rate response to the guidance error Δy^{**} is given by

$$\dot{r} = \frac{1}{\tau_{ap}} (K_{ap} \Delta y^{**} - r)$$

The compensated guidance error output to the autopilot/airframe is

$$\Delta y^{**} = \Delta y^{*} + K V_N$$

where

$$\Delta y^{*} = \Delta y + \xi$$

$$\dot{\xi} = -\tau_2 \xi + (\tau_1 - \tau_2) y$$

$$K = \frac{2.0}{(K_{ap} R_T)}$$

The calculated error signal inputs are

$$\Delta y = \sigma (R_M{}^2 - R_p{}^2)^{1/2}$$

$$\sigma = \mathrm{sgn} (x_M y_T - x_T y_M)$$

$$V_N = V_T \cos (\psi_T + \phi)$$

The commanded lateral acceleration is restricted in magnitude not to exceed the maximum lateral maneuvering limit

$$\left| r\ V_M \right| \leq n_{LIM}$$

Other missile flight path constraints are considered in the model. Missile flight paths are terminated as target misses when any of the following conditions are attained:

- $t \geq t_{MAX}$ flight time exceeds a specified guidance time (jetavator control termination due to sustainer motor termination)
- $l \geq l_{MAX}$ guidance wire payout exceeds maximum wire length.

AUTOPILOT/AIRFRAME TRANSIENT RESPONSE

A discussion of the autopilot/airframe transient response of the simulated missile system to various tank motion inputs is in order as it is critical to the development of the tank evasive maneuvering strategies. In order to automate the guidance mode for the upgraded threat missile it was necessary to design specific compensation networks to stabilize the normally unstable response of "pure" command to L.O.S. guidance. Both the L.O.S. error and rate error compensators, which appear in Fig. 7, were designed by a computer simulation procedure using the TOW missile autopilot compensator design as a guide. The TOW system is described in Ref. 4.

The progression of the compensator design is summarized by the sample response histories presented in Fig. 8. In the four cases presented the horizontal axis represents range along the L.O.S. from gunner to target. The initial target range is given by the c.g. symbol location.

Case 1 shows the Δy response of the system without any compensation, viz. $\Delta y^{**} = \Delta y$. The target is stationary (V_T=0) and the missile is launched with boresight error δ=0.2° relative to the L.O.S. The boresight error convention is specified in Fig. 9. The unstable response of the missile results in a miss of over 300 ft.

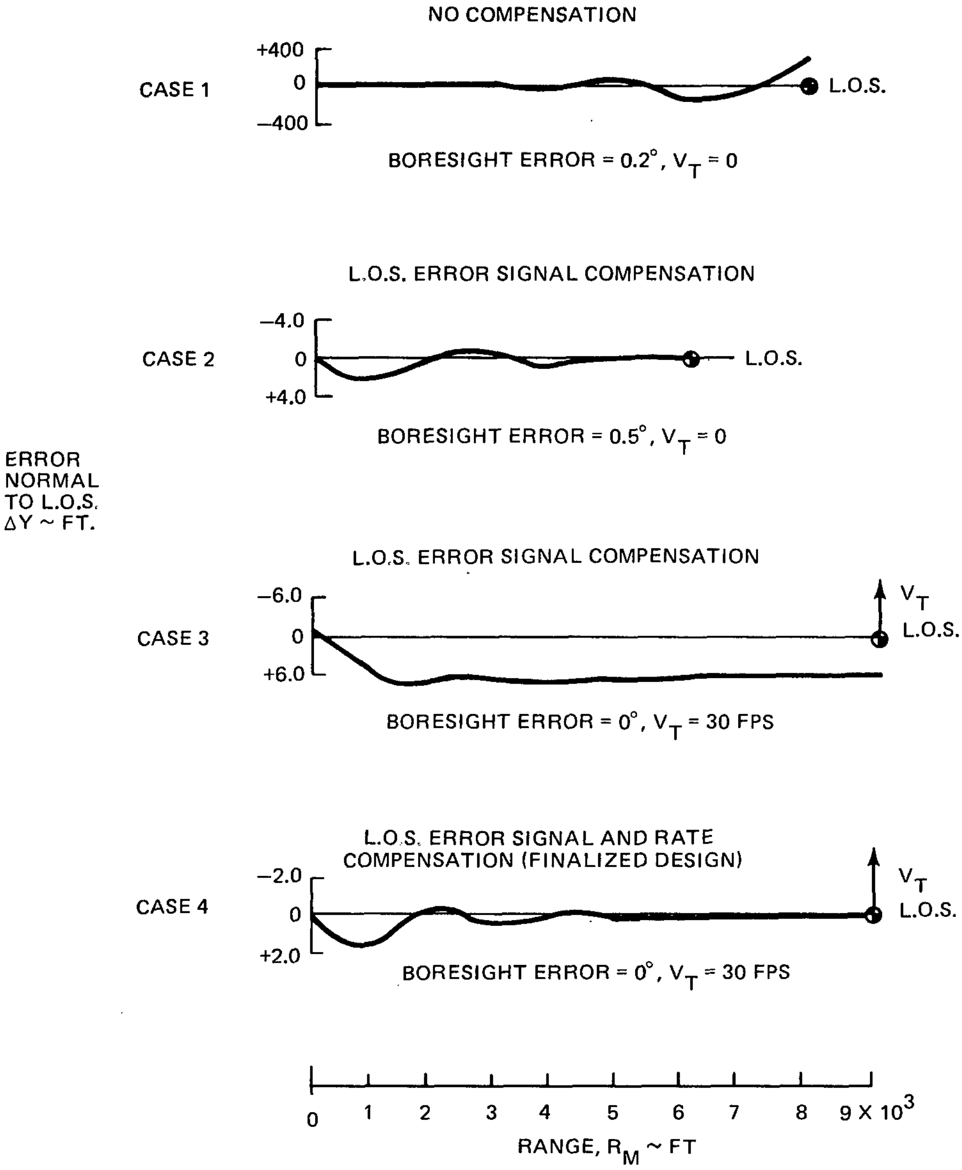


Fig. 8 Missile System Transient Response

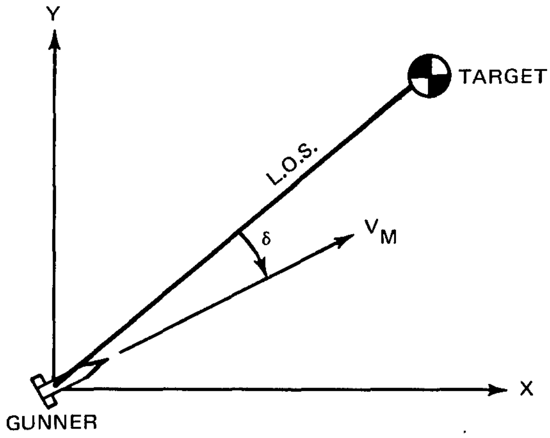


Fig. 9 Boresight Error Convention

Case 2 shows the response with only L.O.S. error compensation ($\Delta y^{**} = \Delta y^{*}$) for a stationary target and shows a direct hit.

Case 3 illustrates the result using only the L.O.S. compensator with the target traveling in a straight line at 30 fps.

Case 4 shows the response to a moving target of the finalized autopilot design ($\Delta y^{**} = \Delta y^{*} + K_1 V_N$). The response as shown in case 4 is representative of that observed by automated systems such as the TOW (See Ref. 4) for similar tank inputs.

Figure 10 illustrates the missile response to tank acceleration/deceleration inputs for the finalized design.

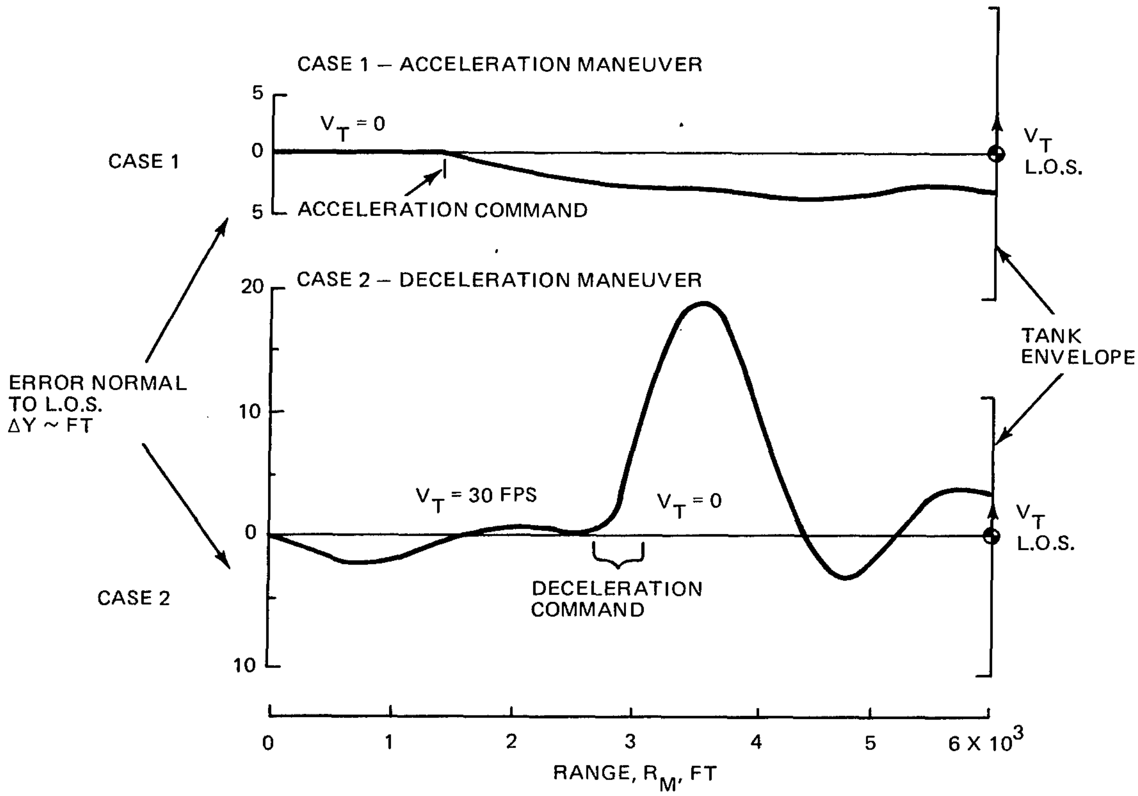


Fig. 10 Missile System Response to Acceleration/Deceleration Inputs

Case 1 shows the Δy response to a sustained maximum acceleration input initiated from rest at the range indicated. The tank envelope length is scaled at the right. Case 2 illustrates the response to a maximum braking deceleration input initiated at a 30 fps velocity condition and sustained until the tank is brought to rest as indicated by the relative range tick marks. The specific gains and time constants associated with the compensators and autopilot/airframe model along with the tank maximal acceleration/deceleration performance parameters are given in the model data portion of section 5 of this report. One can expect from the foregoing development that the deceleration maneuver employed in a "properly timed" manner is important for the eventual evasive strategy development. This conjecture is clearly evident in the computational results shown later.

WARHEAD LETHALITY MODEL

The warhead is assumed detonated by contact with the exterior envelope of the tank vehicle. Based upon the actual contact point and obliquity of the contact, a kill probability is determined from empirical vulnerability data. In these first applications of the methodology, the following simplifications have been introduced: 1) a rectangular exterior envelope (top view) 2) a kill probability of unity for contact anywhere on the exterior envelope independent of obliquity, and 3) a kill probability of zero for an envelope miss independent of miss distance. This simplified kill determination case is important since for this condition the entire burden of maximizing survivability is placed upon the maneuver capability of the vehicle without any vehicle armor considerations.

The development given below describes the more general impact point and impact obliquity calculations for the rectangular envelope made available in the actual computer program. (These would be necessary for use with more extensive terminal vulnerability data.) The coordinate nomenclature is given in Fig. 11. Dimensions of the rectangular tank envelope are specified by the constants A_1, A_2, and B. The $\hat{x}$, $\hat{y}$ coordinate axes centered on the tank provide the basis for the resolution of the relative motion of missile and tank to determine the impact point and obliquity.

The missile velocity vector components in the $\hat{x}$, $\hat{y}$ tank coordinates are

$$V_{M_{\hat{x}}} = V_M \cos (\gamma + \phi)$$

$$V_{M_{\hat{y}}} = V_M \sin (\gamma + \phi)$$

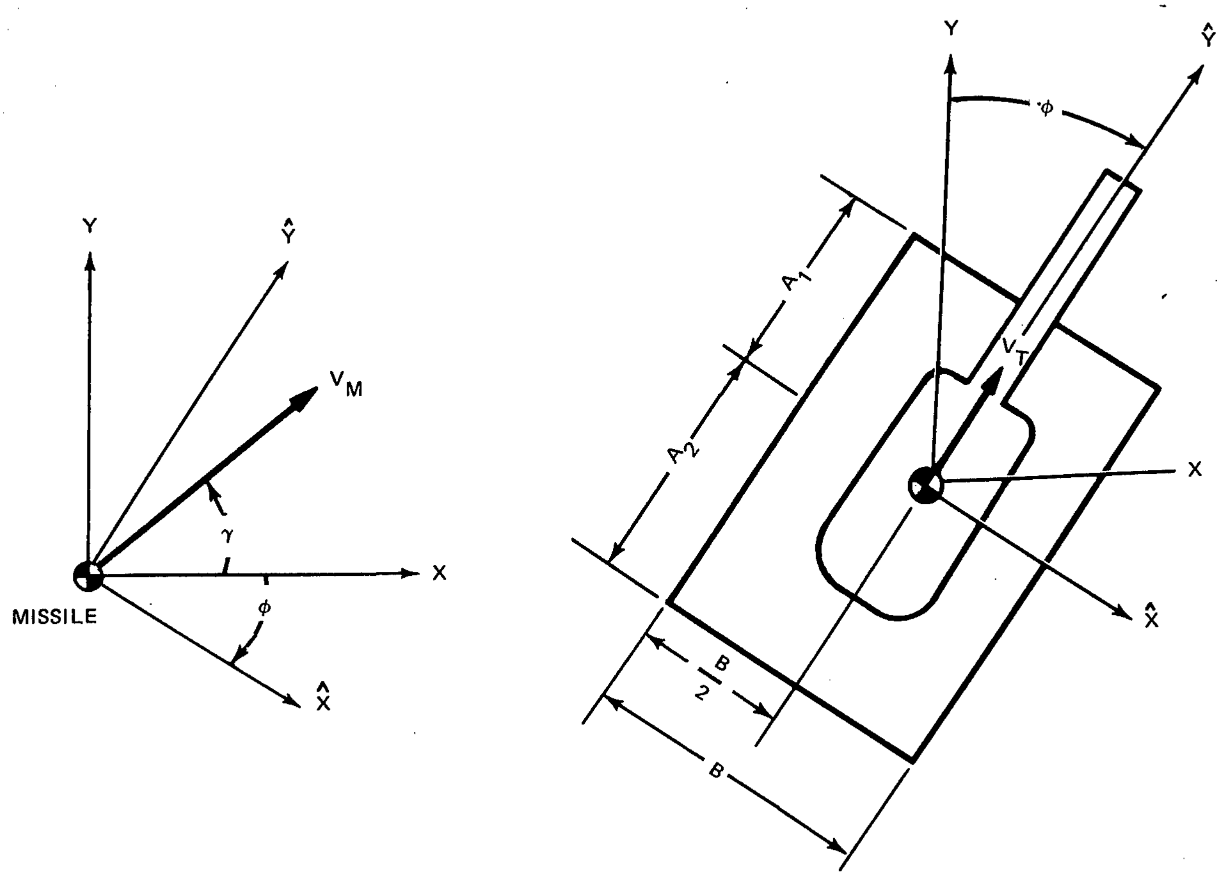


Fig. 11 Tank Relative Coordinate System $\hat{X}$, $\hat{Y}$

Hence, the relative velocity components of the missile in the $\hat{x}$, $\hat{y}$ tank coordinates are

$$V_{R_{\hat{x}}} = V_{M_{\hat{x}}}$$

$$V_{R_{\hat{y}}} = V_{M_{\hat{y}}} - V_T$$

Figure 12 gives the envelope side convention adopted for the impact and obliquity calculations. The relative velocity V_R is drawn emanating from the missile coordinates $\hat{x}_R$, $\hat{y}_R$ relative to the tank. These coordinate values correspond to the last calculated position during the trajectory integration for which the relative range rate, $\dot{R}$, first becomes positive.

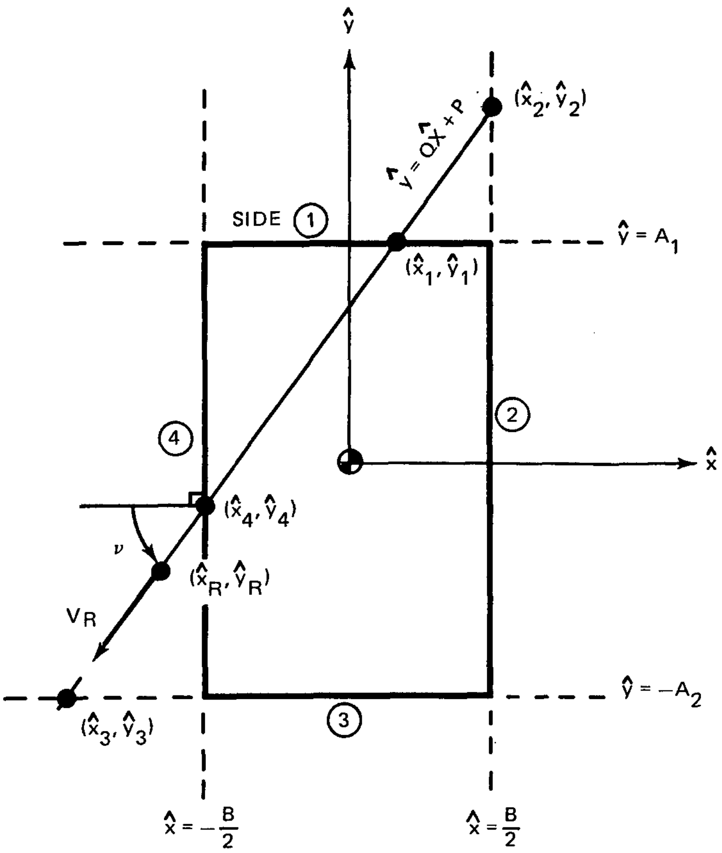


Fig. 12 Tank Envelope Side Convention

The relative trajectory near termination, given $\hat{x}_R$, $\hat{y}_R$ and V_R, is approximated as a straight line $\hat{y} = Q\hat{x}+P$ with

$$Q = \frac{V_{R_{\hat{y}}}}{V_{R_{\hat{x}}}} \quad \text{and } P = \hat{y}_R - Q\hat{x}_R$$

The intersections of the "extended" envelope sides (shown by the dashed lines) and the relative trajectory can be computed from the following conditions:

Extended Side 1
$$\begin{aligned} \hat{y} &= Q\hat{x}+P \\ \hat{y} &= A_1 \end{aligned} \quad \Longrightarrow \quad \hat{x}_1 = \frac{A_1 - P}{Q} \quad , \hat{y}_1 = A_1$$

2
$$\begin{aligned} \hat{y} &= Q\hat{x}+P \\ \hat{x} &= \frac{B}{2} \end{aligned} \quad \Longrightarrow \quad \hat{x}_2 = \frac{B}{2} \ , \ \hat{y}_2 = P + Q\frac{B}{2}$$

③ $\hat{y} = Q\hat{x}+P$, $\hat{y} = -A_2$ $\Longrightarrow$ $\hat{x}_3 = -\left(\frac{A_2+P}{Q}\right)$, $\hat{y}_3 = -A_2$

④ $\hat{y} = Q\hat{x}+P$, $\hat{x} = -\frac{B}{2}$ $\Longrightarrow$ $\hat{x}_4 = -\frac{B}{2}$, $\hat{y}_4 = P - \frac{QB}{2}$

The "actual" side intersections can then be determined from the following test sequence:

If $(-B/2 < \hat{x}_1 < B/2)$ then side ① is intersected.

If $(-A_2 \leq \hat{y}_2 \leq A_1)$ then side ② is intersected.

If $(-B/2 < \hat{x}_3 < B/2)$ then side ③ is intersected.

If $(-A_2 \leq \hat{y}_4 \leq A_1)$ then side ④ is intersected.

The test above leads to at most two side intersection candidates labeled side I and II, determined by the order of test sequence satisfaction. The actual impact side and impact point computation for a single side intersection candidate is obvious. The actual impact side (and impact point) for the case of two candidates is as follows: Let $\hat{x}_I$, $\hat{y}_I$ and $\hat{X}_{II}$, $\hat{y}_{II}$ be the side intersection coordinates resulting from the above test. Form the components of vector T

$$T_{\hat{x}} = \hat{x}_{II} - \hat{x}_I$$

$$T_{\hat{y}} = \hat{y}_{II} - \hat{y}_I$$

Construct the inner product

$$T \cdot V_R = T_{\hat{x}} V_{R_{\hat{x}}} + T_{\hat{y}} V_{R_{\hat{y}}}$$

If $T \cdot V_R > 0$ then side I is the impact side

$T \cdot V_R < 0$ then side II is the impact side

The obliquity of the impact, given by angle ν in Fig. 12, is computed by

$$\nu = \frac{\pi}{2} - \left| \tan^{-1} \frac{V_{R_{\hat{y}}}}{V_{R_{\hat{x}}}} \right| \quad \text{for impact on side 1 or 3}$$

$$\nu = \left| \tan^{-1} \frac{V_{R_{\hat{y}}}}{V_{R_{\hat{x}}}} \right| \quad \text{for impact on side 2 or 4}$$

4. COMPUTATIONAL APPROACH

The computational method is constructed in two phases: a "learning" phase, which is associated with the optimal strategy development for the tank, and a statistics phase associated with the determination of the tank survivability and missile effectiveness measures.

LEARNING PHASE

Strategy Initialization

A stochastic learning algorithm has been developed for strategy resolution in "two-player" one-on-one duels in Ref. 1. That methodology has been applied to a "one-player" aircraft/missile avoidance problem in a similar manner as described in Ref 2. In this setting the aircraft had its strategy resolved in accordance with a survival goal while the missile employed a prespecified guidance policy. The tank/anti-tank missile problem is resolved in a fashion analogous to the aircraft/missile problem of Ref 2.

One initially begins with the tank strategy represented by Table 2.

TABLE 2 TANK VEHICLE DECISION TABLE

		ELEMENTAL CONTROLS U_j				
		1	•	•	•	5
	1	•	•	•	•	•
	•	•	•	•	•	•
OBSERVABLE REGIONS	•	•	•	p_{ij}	•	•
	•	•	•	•	•	•
x_i	•	•	•	•	•	•
	N	•	•	•	•	•

In this table x_i, i=1, . . ., N represent the observable set decomposition given in Fig. 3, (N=132), and u_j j=1, . . ., 5 represent the elemental maneuver choices as shown in Fig 4 and Table 1. Initially the table begins with p_{ij} = 0.2 for all i=1, . . ., N and j=1, . . ., 5; that is, the elemental control choices are selected at random in an equally likely manner for each visited observable region x_i.

Trajectory and Outcome Simulation

With the initial strategy for the tank selected as given previously, the simulation of a trajectory, outcome determination, and strategy modification cycle of the learning phase are begun. The trajectory simulation begins with an initial condition, relative range R, and angle-off ω, being chosen at random in a uniform manner over the allowable space of initial conditions (the boresight angle $\delta = 0^\circ$ in all simulations). A control choice u_j is then selected for the tank in accordance with the assumed starting strategy for the specific region x_i given by the range (R) and angle-off (ω) thresholds.

The missile and tank trajectories are then integrated until the next tank threshold is crossed and a new tank control decision selected. This process continues until one of the following events occurs: a minimum miss distance (R_{MIN}) is obtained, a missile guidance time limit (t_{MAX}) is exceeded, or a missile guidance wire length limit (l_{MAX}) is exceeded. The region/control sequence employed by the tank is temporarily stored for reference.

The outcome of any trajectory sample is determined in the following manner: If $R_{MIN} > 100$ ft or $t > T_{MAX}$ or $R_M > L_{MAX}$ a miss is recorded; for $R_{MIN} < 100$ ft a tank envelope contact test is performed. For the simplified warhead/tank interaction model considered, envelope contact at any point results in a kill (P_K=1.0) and no envelope contact a miss (P_K=0.0).

Strategy Modification

For each of the two outcomes (ρ) we employ the following weightings $\mu(\rho)$ in the strategy modification portion of the learning phase.

Outcome ρ	μ (ρ)
Kill	½
Miss	2

These weightings are consistent with a survival rationale as a goal in the combat for the tank. If during a sample trajectory the tank visits region x_i using control u_k and ultimately outcome ρ occurs, then the strategy table is modified from p_{ij} to $\bar{p}_{ij}$ by first modifying p_{ij} to p_{ij}^* where

$$p_{ij}^* = \mu(\rho) p_{ij} \qquad j = k$$

$$p_{ij}^* = p_{ij} \qquad j \neq k$$

The p^*_{ij} is then renormalized to form $\overline{p}_{ij}$

$$\overline{p}_{ij} = \frac{p^*_{ij}}{\sum\limits_j p^*_{ij}}$$

This process is carried out for only those x_i, u_j pairs temporarily stored during that trajectory. This results in an updated strategy table for the tank.

Approximately 100 trajectories with initial conditions selected at random within each region are simulated to obtain a converged decision table that represents the "optimized" survival strategy for the tank.

STATISTICS PHASE

The statistics phase of the computations now fixes the converged decision table and computes the tank survivability and missile effectiveness measures in Monte Carlo fashion. For the survivability data approximately 100 trajectory computations with initial conditions in each observable region were obtained. The quantitative measure of survivability is the complement of the missile kill probability as a function of the range (R) and angle-off (ω) thresholds relative to the tank. To facilitate the presentation of results the missile kill probability interpretation has been adopted.

5. COMPUTATIONAL RESULTS

The computational results obtained during this program development fall into three study categories. Study No. 1 establishes the baseline survivability results and strategies against which tank and missile model parameter variations can be quantified. Study No. 2 examines an extension of the baseline tank observable set (comprised of thresholds of relative range R, and angle off ω) to include tank speed thresholds. Study No. 3 examines the sensitivity of the survivability to a change in the braking deceleration performance from the baseline tank configuration. The model data as employed in each of the study categories are described.

MODEL DATA

The tank and missile data as employed in the three studies reported are presented in Table 3. In all studies the tank acceleration parameter a^+ was modeled as shown in Fig 13. The slopes of the four straight line segments in the speed/time plot approximate the a^+ function of a representative tank vehicle. In study No. 1 the warning system or observable set model comprised 11 x 12 = 132 regions as shown in Fig 3. In studies 2 and 3 the observable set comprised 11 x 12 x 5 = 660 regions. (There are five tank velocity intervals; 0-10 fps, 10-20 fps, 20-30 fps, 30-40 fps, and 40-50 fps,within each R, ω region specification.)

STUDY NO. 1 - BASELINE CONFIGURATION

The tank survivability results are presented in Fig 14. Four levels of P_K have been arbitrarily selected to simplify the results presentation. The actual P_K values, however, are available in the computational results for more detailed appraisals. The diagrams show the kill probability of the missile for all launch conditions from 250 m to 3.0 km in range at all angles-off in the tank-centered observable space. Missiles were not launched at relative ranges $R < 250$ m because of the usual transient response characteristics degrading operational performance; nor were they launched at ranges $R > 3.0$ km because of guidance wire length and flight duration limits. In both the non maneuvering and maneuvering cases the tank velocity at missile launch is $V_T(0) = 30$ fps. The results on the left show the survivability for the non maneuvering case. This is

TABLE 3. TANK AND MISSILE MODEL DATA

Data	TANK VEHICLE												MISSILE								
	Dynamical Parameters					Warning System			Envelope/armor				Autopilot/Airframe Parameters				System Constraints			Velocity Profile	
Study No.	K_1	K_2	a+	a-	$V_T(0)$	Observables	Threshold Decomposition	Min-Max Range of Observables	A_1	A_2	B	P_K	K_{ap}	τ_{ap}	τ_1	τ_2	Guide Limit t_{MAX}	Wire Limit L_{MAX}	Lat Acc Limit n_{LIM}	$V_M(0)$	α
	rad/sec	ft/sec^2	ft/sec^2	ft/sec^2	fps				ft	ft	ft		1/ft. sec	sec	sec	sec	sec	ft	ft/sec^2	fps	ft/sec^2
1	0.5	8.05	See Fig. 13	-32.2	30	R ω	11 divisions 12 divisions	(0-3KM) (0-360°)	10.5	11.5	12.0	Rect. Envelope Hit P_K = 1.0 Miss P_K = 0.0	0.008	0.6	0.3	2.5	27.5	10,200	45	360	1.0
2	0.5	8.05	See Fig. 13	-32.2	(0-45)	R ω V_T	11 divisions 12 divisions 5 divisions	(0-3KM) (0-360°) (0-50 fps)	10.5	11.5	12.0	Rect. Envelope Hit P_K = 1.0 Miss P_K = 0.0	0.008	0.6	0.3	2.5	27.5	10,200	45	360	1.0
3	0.5	8.05	See Fig. 13	-16.1	(0-45)	R V_T	11 divisions 12 divisions 5 divisions	(0-3KM) (0-360°) (0-50 fps)	10.5	11.5	12.0	Rect. Envelope Hit P_K = 1.0 Miss P_K = 0.0	0.008	0.6	0.3	2.5	27.5	10,200	45	360	1.0

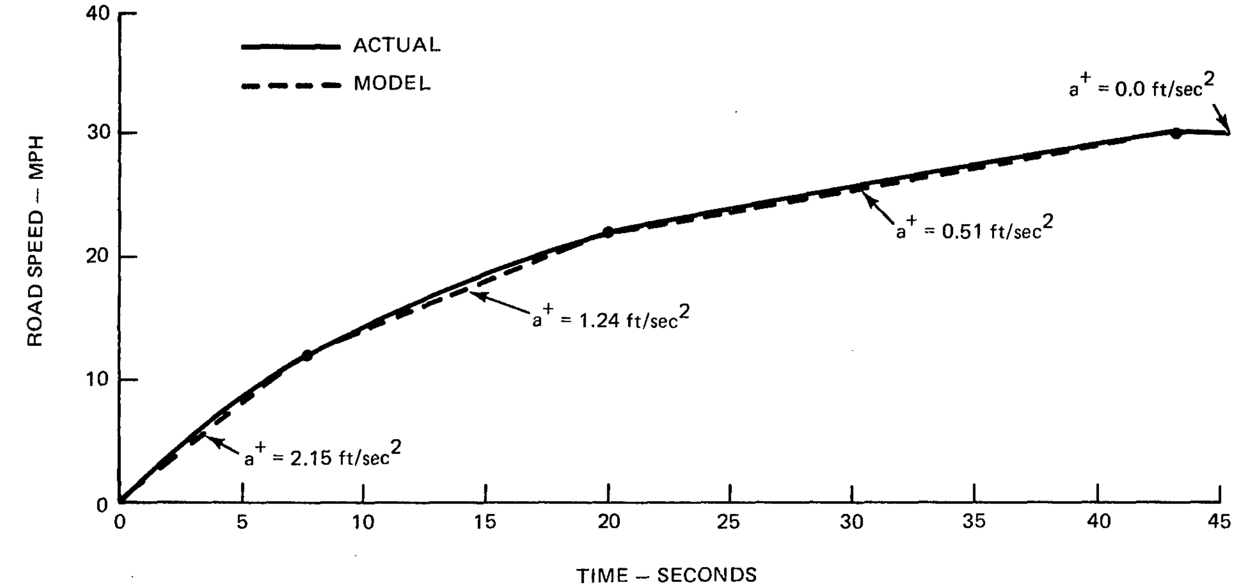


Fig. 13 Tank Forward Acceleration Characteristics

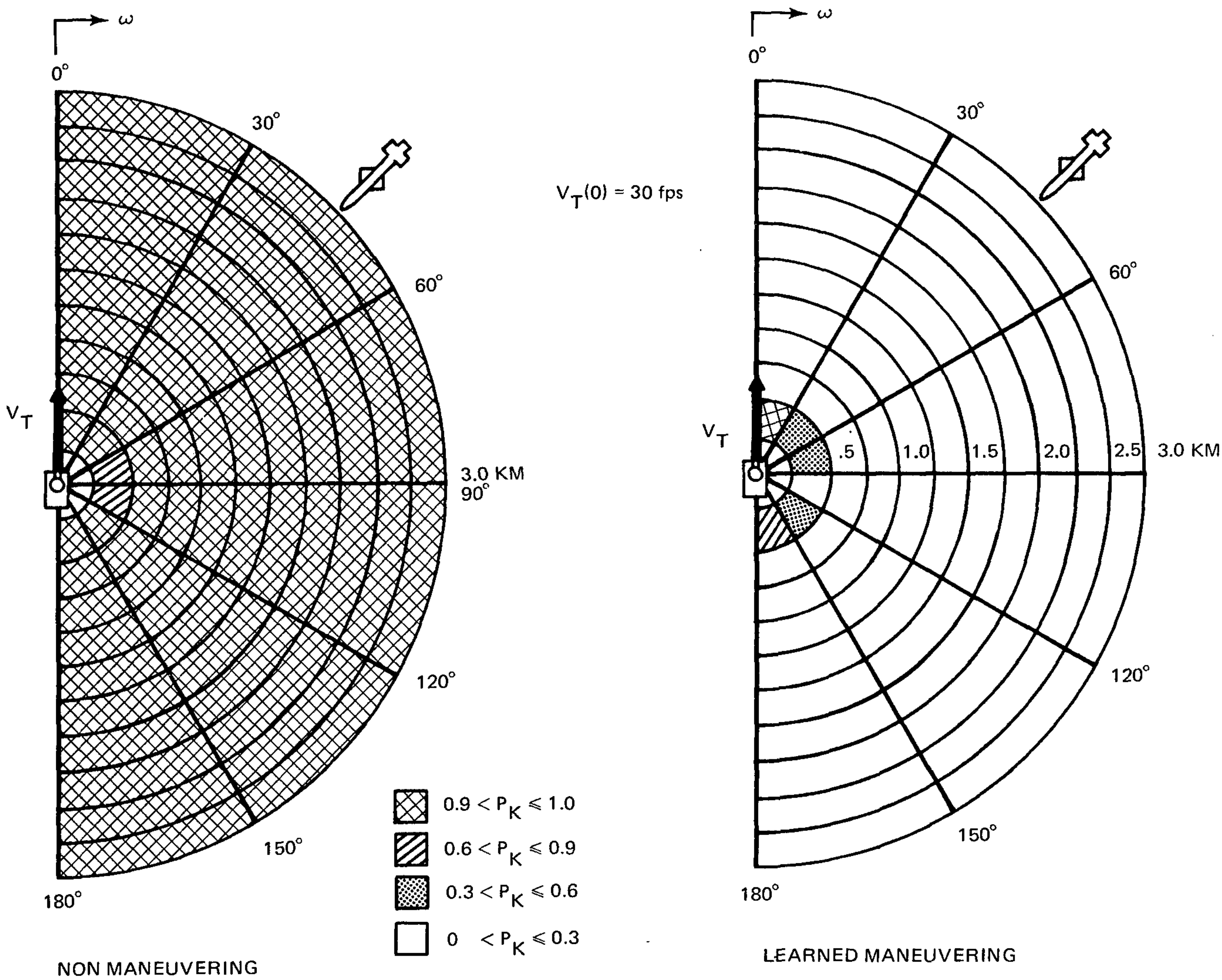
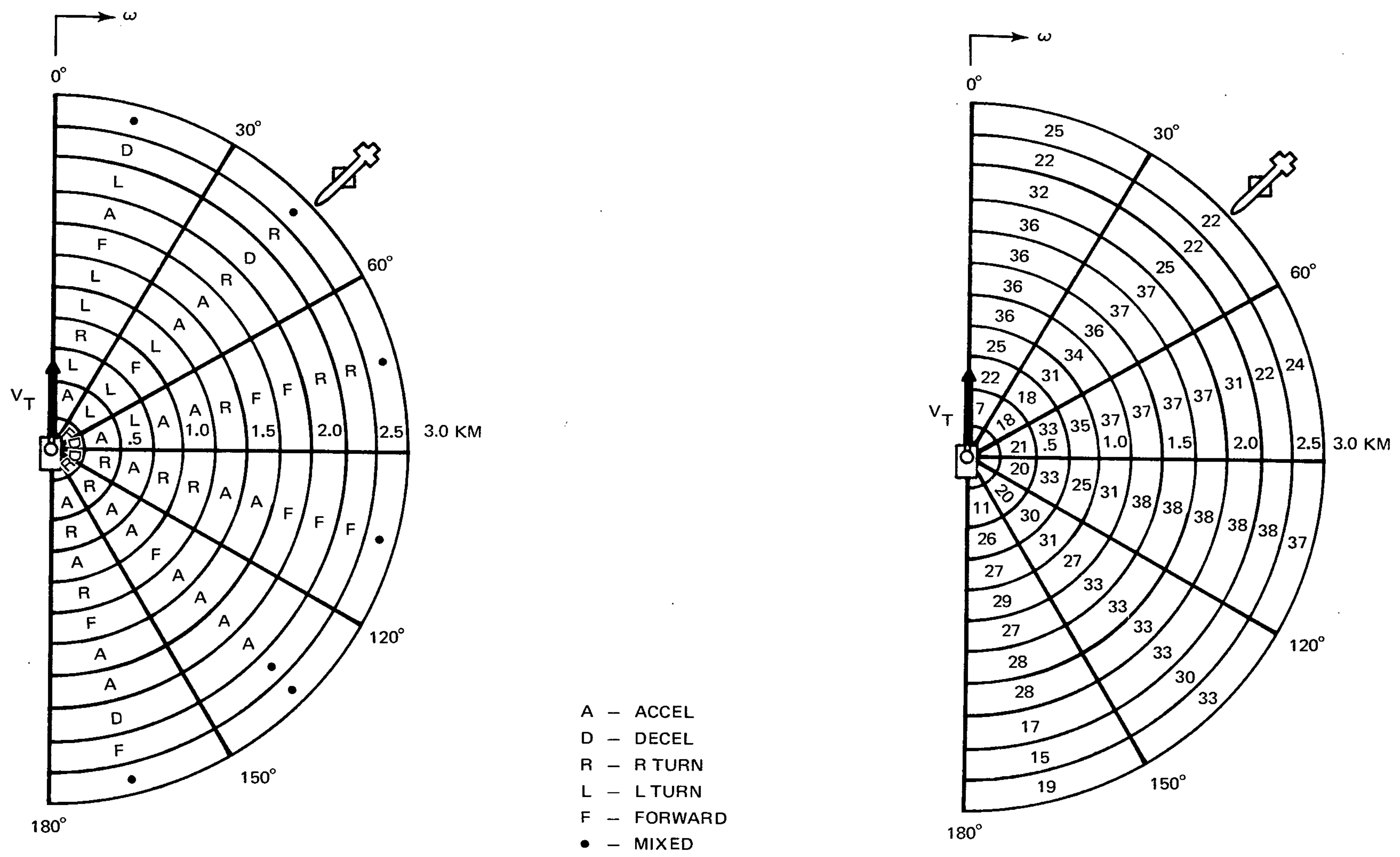


Fig. 14 Tank Survivability Results (Study No. 1)

equivalent to the case where the tank does not have any threat warning information with which to effect evasive maneuvers. In this case the tank maintains a constant direction and speed during the engagement. The diagram at right shows the improvement in survivability achieved with optimal learned maneuvering.

The optimized tank strategy and average minimum miss distance statistics corresponding to the learned maneuver survivability results are shown in Fig 15. The key control decisions in the optimal strategy are the two deceleration maneuvers in the $\omega = 60$ to 120° orientations within the 250m range threshold. The overall optimal strategy is composed of tank maximal performance turns to achieve a beam ($\omega = 60$ to 120°) orientation followed by a maximal deceleration at the R = 250m range threshold. This basic strategy provides the average miss distance statistics shown in Fig 15b for all regions of the initial condition space.

This strategy is more easily visualized in illustrations of sample engagement trajectories with the tank employing the optimized strategy shown in Fig 15a. Each trajectory will be viewed from three vantage points; relative to the tank, relative to the gunner L.O.S., and in absolute coordinates. The first of the trajectory samples is shown in Figs. 16, 17, and 18. (These figures are "hard copies" of actual computer graphics displays, with the trajectories enhanced for reporting clarity.) Trajectory No. 1 has launch conditions R = 7000 ft, and $\omega = 5^{\circ}$, relative to the tank as shown in the display captions. Figure 16 shows the resulting trajectory in tank centered coordinates. The tank strategy brings the missile into a beam orientation using turns and then subsequently decelerates the tank to provide the miss of $R_{MIN} = 37$ ft given in the caption. (Due to scaling the miss cannot be distinguished in the figure.) The missile elapsed flight time to the R_{MIN} point is given by the time 18.2 sec. Figure 17 shows the same result when viewed along the gunner L.O.S. The range from the tank is given along the abscissa, and the error normal to the L.O.S. (Δy) given along the ordinate. The efficiency of a properly timed deceleration maneuver in forcing a missile overshoot is readily apparent. In this case the deceleration is initiated at the 250m (825 ft) or closest range threshold to the tank vehicle. Figure 18 shows the same trajectory in absolute coordinates. The scale has been adjusted to show the tank trajectory in its entirety and only the terminal portion of the missile trajectory. The turning maneuver of the tank to achieve a beam aspect to the oncoming missile is readily observed.



a) OPTIMAL STRATEGY

b) AVERAGE MINIMUM MISS DISTANCE ~ FT

Fig. 15 Optimal Strategies and Miss Distance Statistics (Study No. 1)

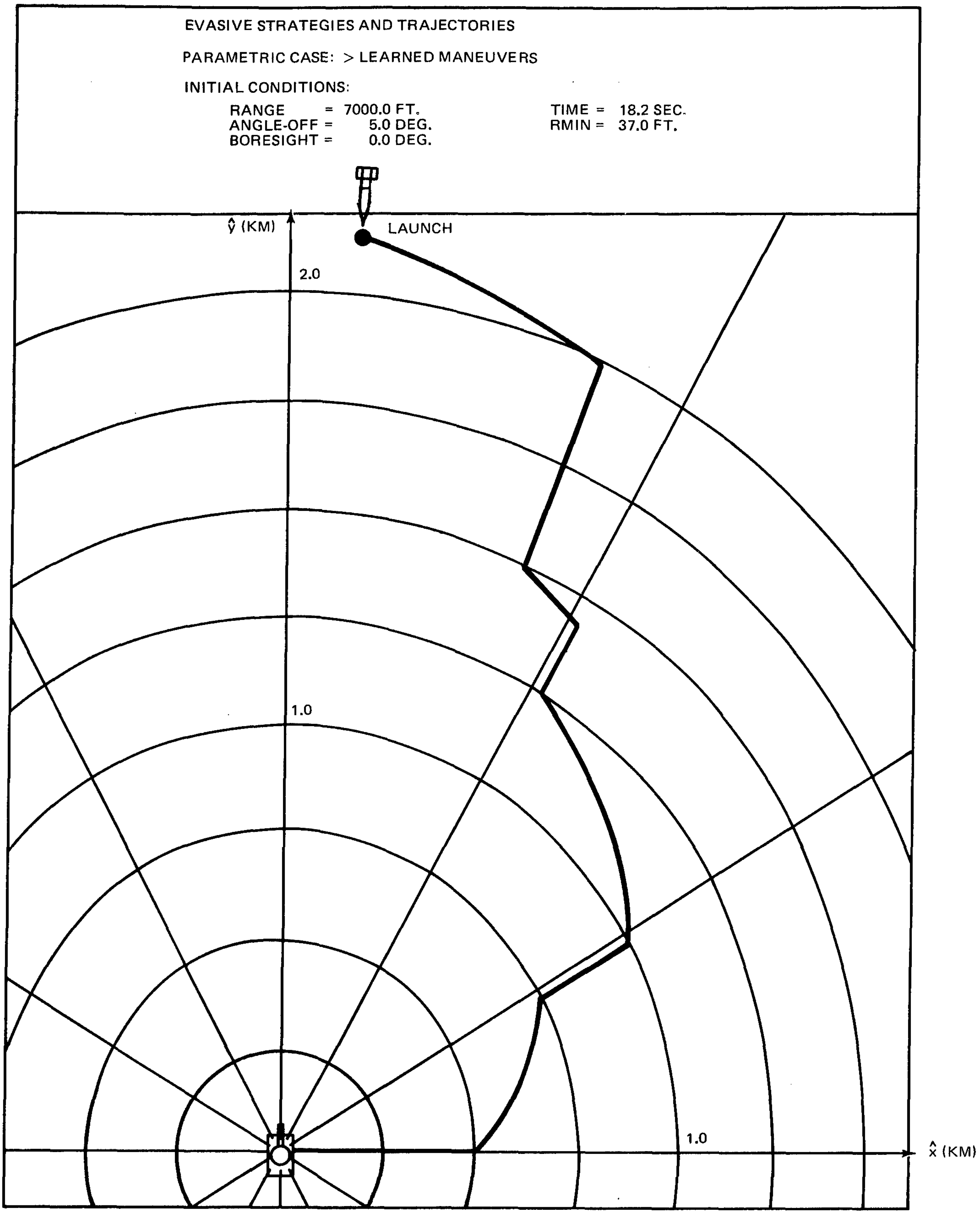


Fig. 16 Sample Trajectory No. 1 (Tank Coordinates)

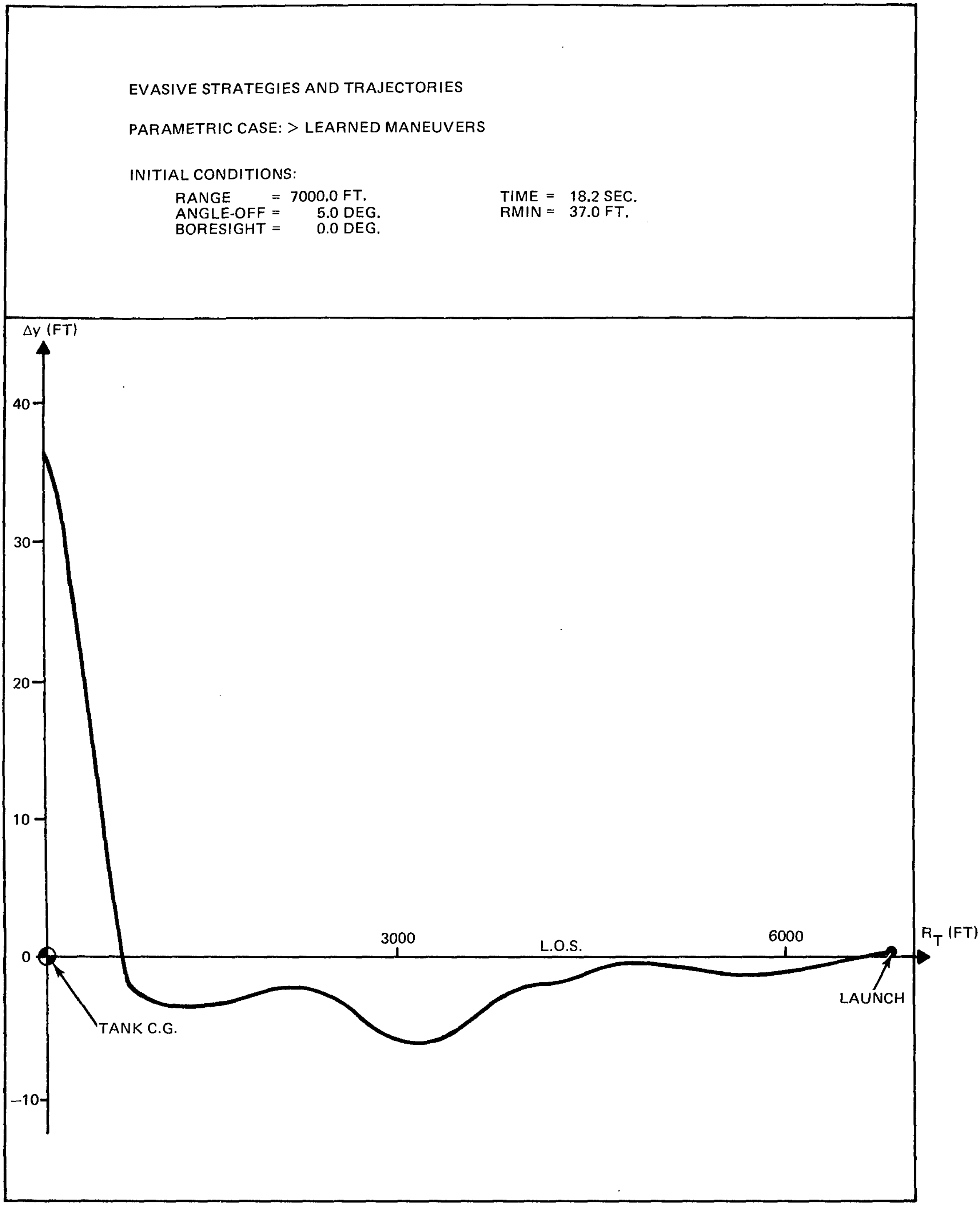


Fig. 17 Sample Trajectory No. 1 (Along Gunner L.O.S.)

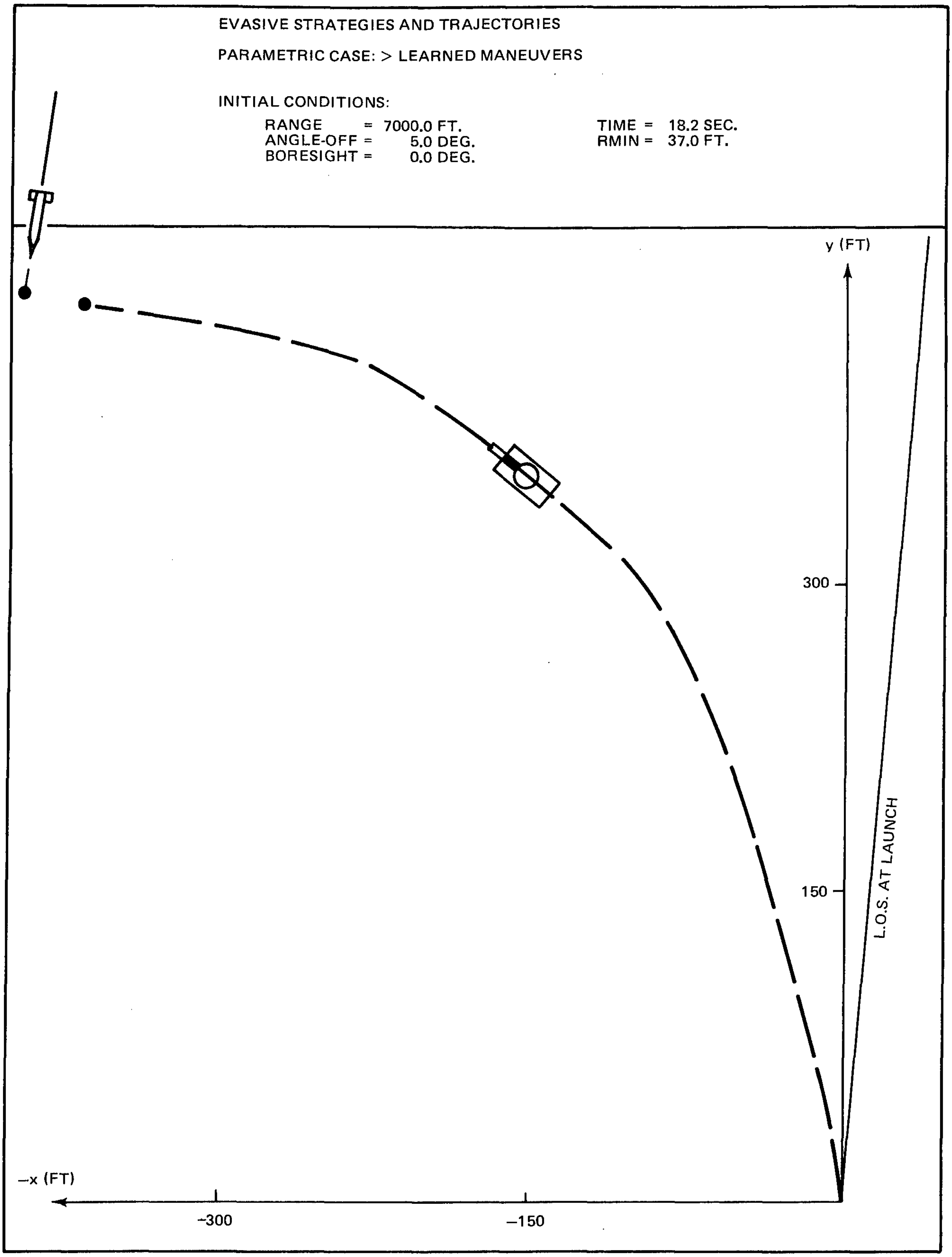


Fig. 18 Sample Trajectory No. 1 (Absolute Coordinates)

Trajectory No. 2 has launch conditions R = 7000 ft, and $\omega = 80^\circ$ relative to the tank. The results viewed in the tank coordinates, along the L.O.S., and in absolute coordinates are given in Figs 19, 20, and 21, respectively. Again, the turn to achieve beam aspect followed by deceleration is the optimal maneuver. The same optimal maneuver characteristics appear in trajectory No. 3 (see Figs. 22-24). In this case the launch range R = 7000 ft and angle-off $\omega = 175^\circ$ indicates a firing from behind the tank in its direction of travel.

STUDY NO. 2. TANK OBSERVABLE SET VARIATION

This study examines the extension of the baseline tank observable set to include tank speed information in the strategy development. As described in Table 3, five intervals of tank speed were assumed for the speed threshold decomposition; they are 0-10 fps, 10-20 fps, 20-30 fps, 30-40 fps, and 40-50 fps. This expands the total number of regions from 132 in the baseline case to 132 x 5 or 660 contingencies or regions for this case, and correspondingly augments the associated decision table shown in Table 2 to N = 660. The survivability results for this case are shown in Fig. 25. The nonmaneuvering case (upper left) shows the kill probability for all tank initial velocities in the 0-45 fps range. The remaining figures indicate the missile effectiveness when the tank employs optimal maneuvering for the initial tank speed in the ranges indicated. To effectively employ the braking strategy which induces a large missile transient error, the tank requires a "reasonable speed" from which to initiate the maneuver. This "speed" depends upon the missile transient response to acceleration inputs, missile flight time to impact, tank envelope size, and tank deceleration capability. If the tank speed at missile launch is relatively low, the vehicle will require its maximum acceleration capability to achieve this "speed" as quickly as possible. The missile effective launch range in this case decreases with increasing tank initial speed. As indicated in the results, the "reasonable speed" is 10-20 fps for the specific tank and missile represented. The optimized strategy again indicates turning maneuvers on the part of the tank for missile launches in the head-on or rear aspect to ultimately achieve a missile beam aspect for initiating the braking deceleration maneuver.

STUDY NO. 3. TANK DECELERATION PARAMETER VARIATION

This study examines the effect of a reduction in braking deceleration capability on the tank vehicle survivability computed in Study No. 2. Several experienced

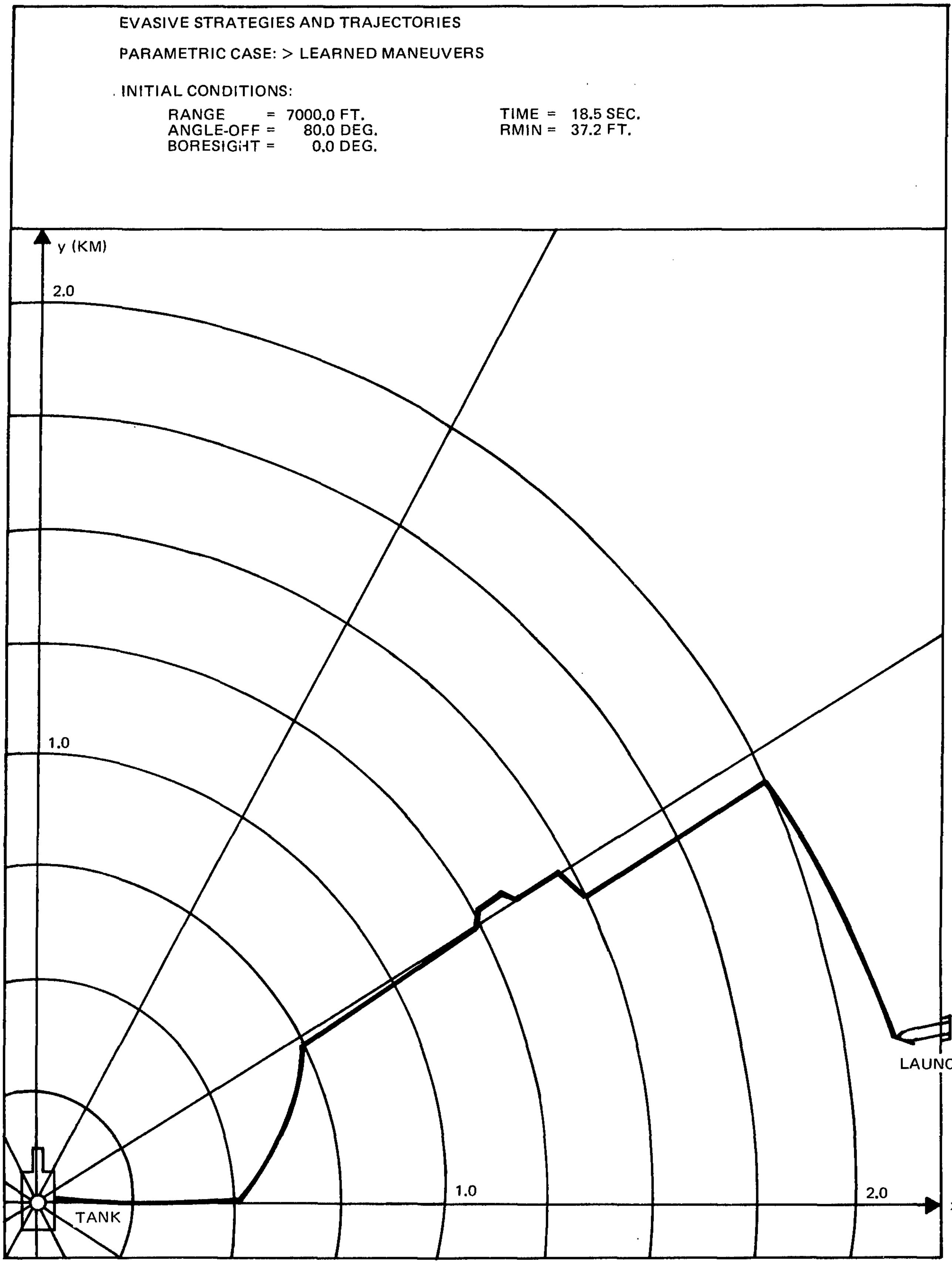


Fig. 19 Sample Trajectory No. 2 (Tank Coordinates)

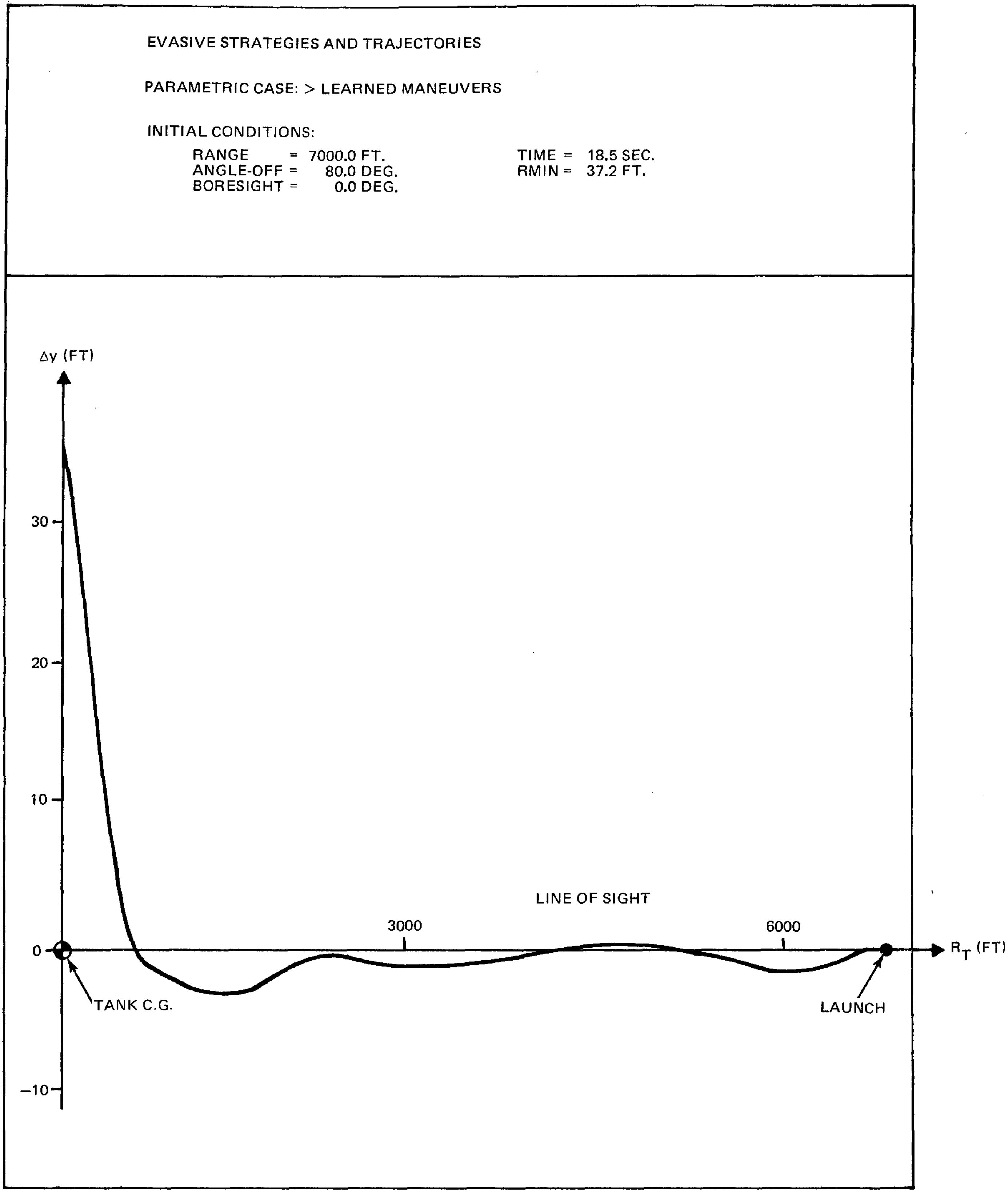


Fig. 20 Sample Trajectory No. 2 (Along Sunner L.O.S.)

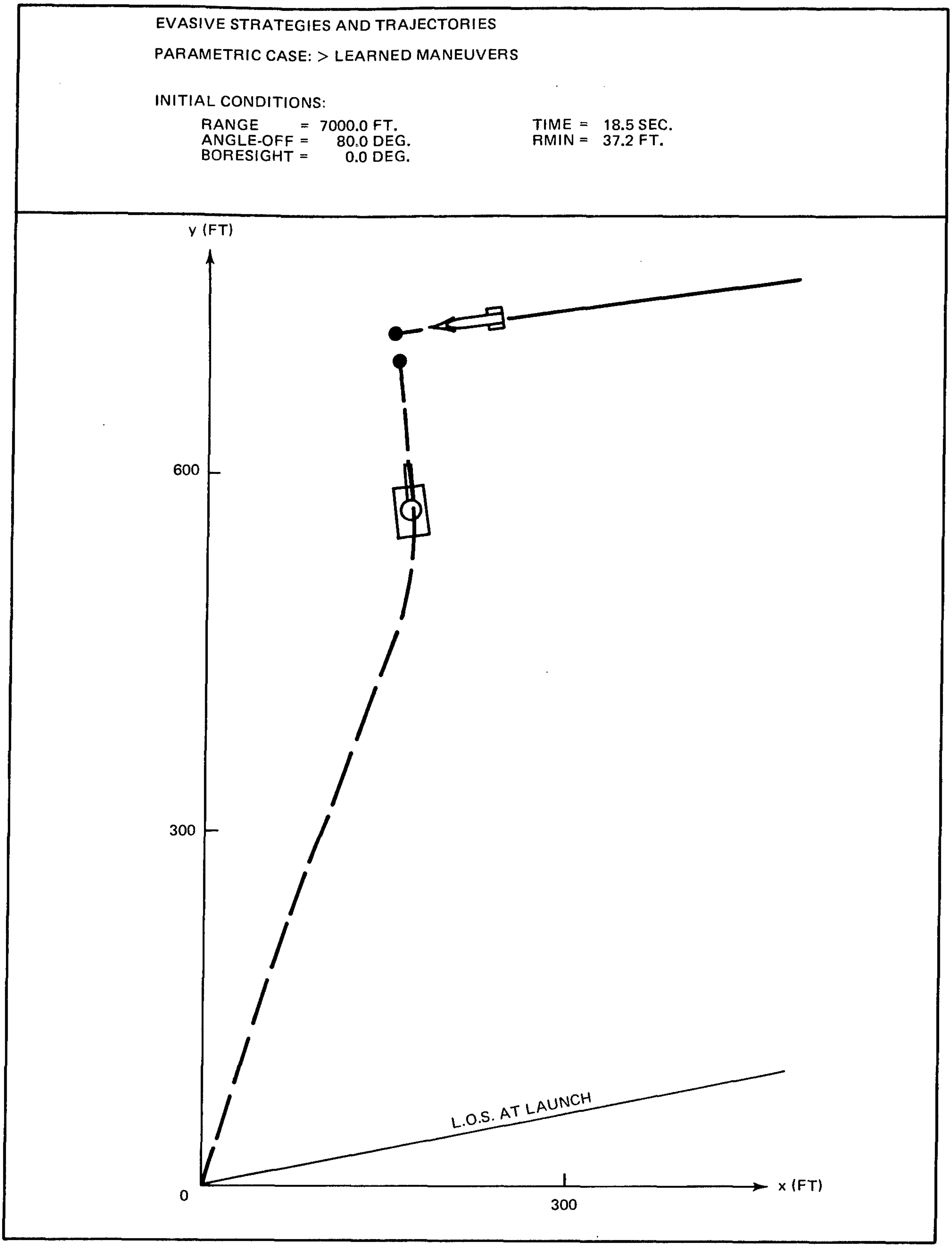


Fig. 21 Sample Trajectory No. 2 Absolute Coordinates

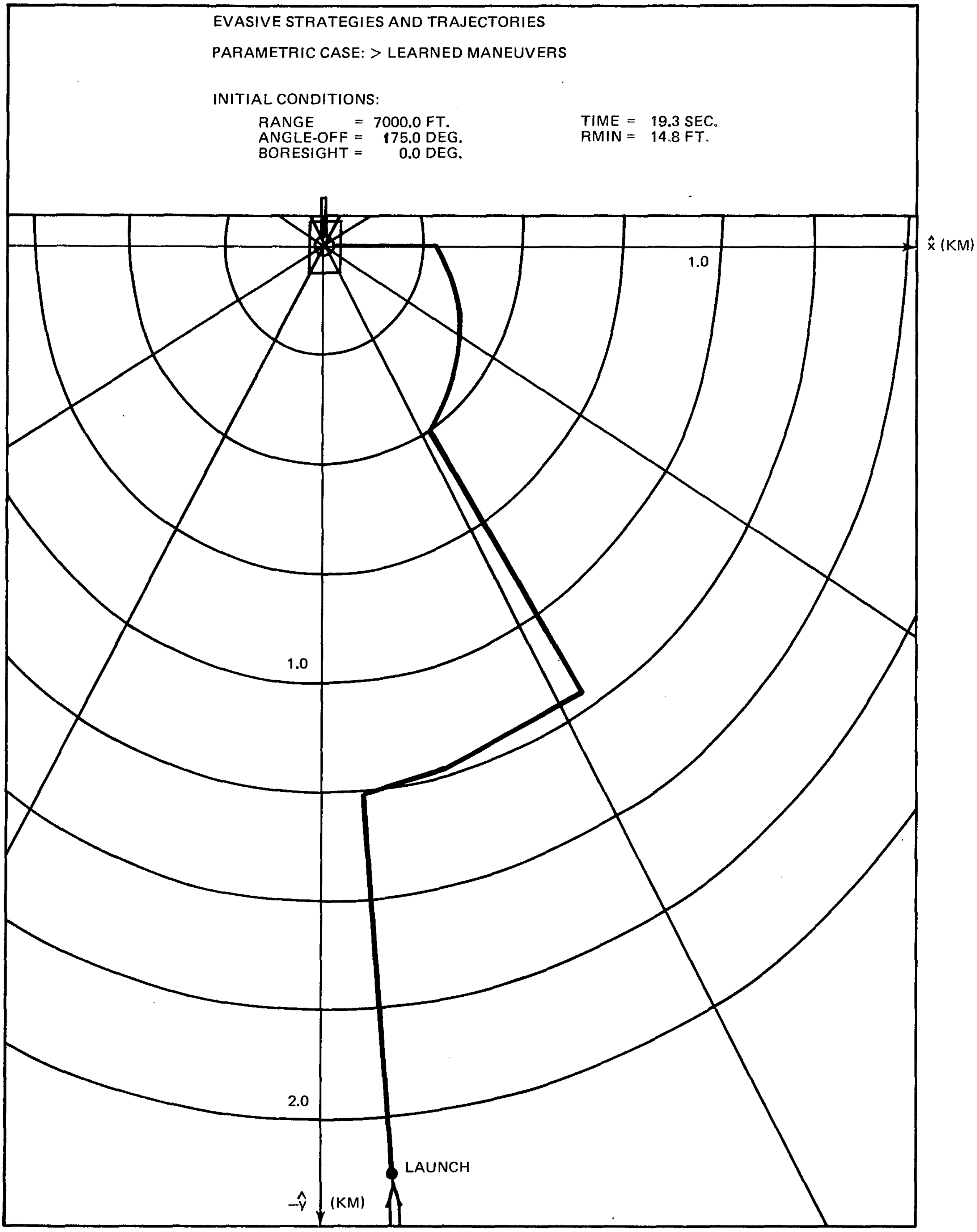


Fig. 22 Sample Trajectory No. 3 (Tank Coordinates)

EVASIVE STRATEGIES AND TRAJECTORIES

PARAMETRIC CASE: > LEARNED MANEUVERS

INITIAL CONDITIONS:

RANGE =	7000.0 FT.	TIME =	19.3 SEC.
ANGLE-OFF =	175.0 DEG.	RMIN =	14.8 FT.
BORESIGHT =	0.0 DEG.		

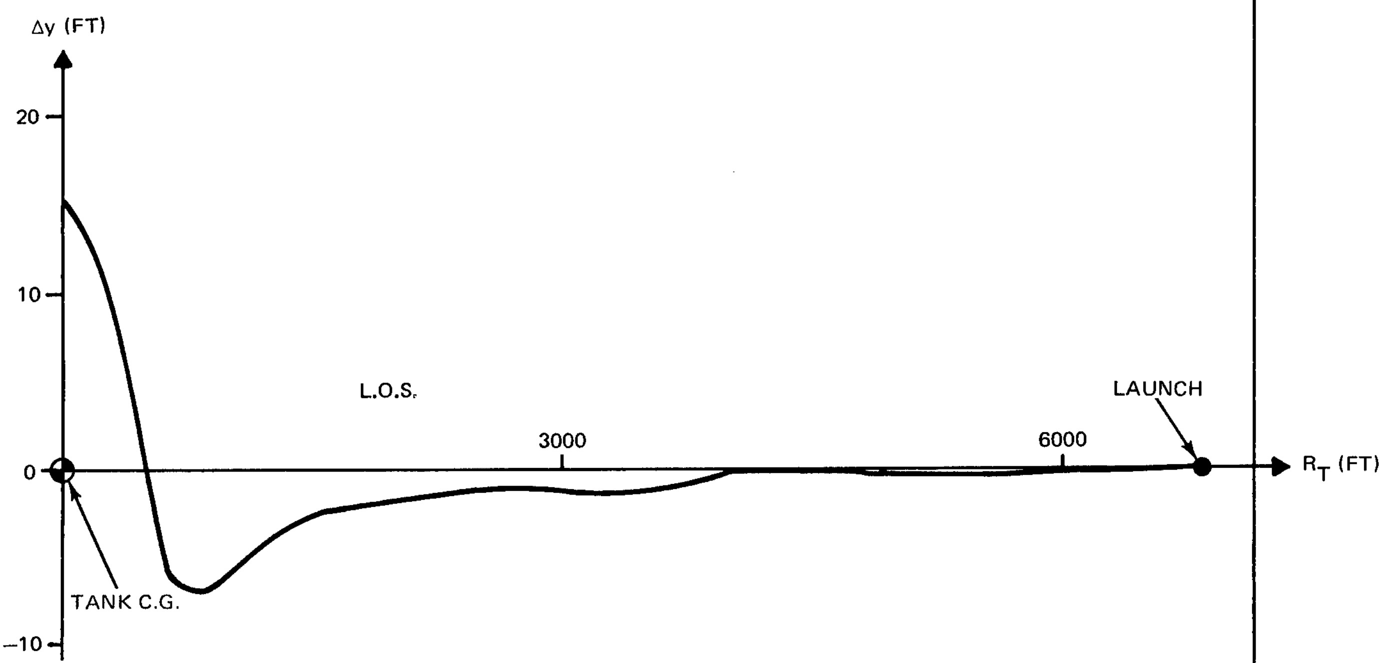


Fig. 23 Sample Trajectory No. 3 (Along Gunner L.O.S.)

EVASIVE STRATEGIES AND TRAJECTORIES

PARAMETRIC CASE: > LEARNED MANEUVERS

INITIAL CONDITIONS

RANGE =	7000.0 FT.	TIME =	19.3 SEC.
HEADING =	175.0 DEG.	RMIN =	14.8 FT.
BORESIGHT =	0.0 DEG.		

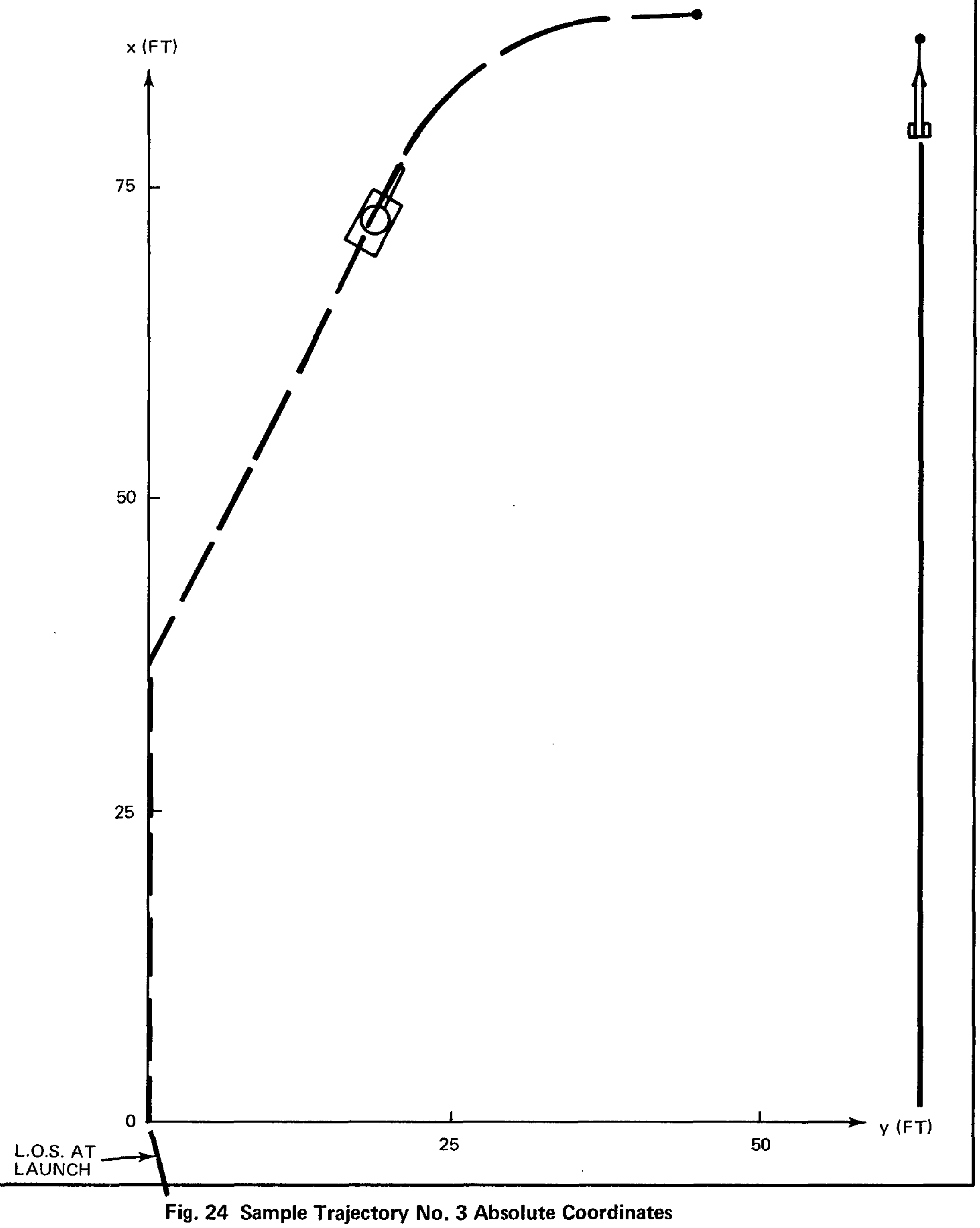


Fig. 24 Sample Trajectory No. 3 Absolute Coordinates

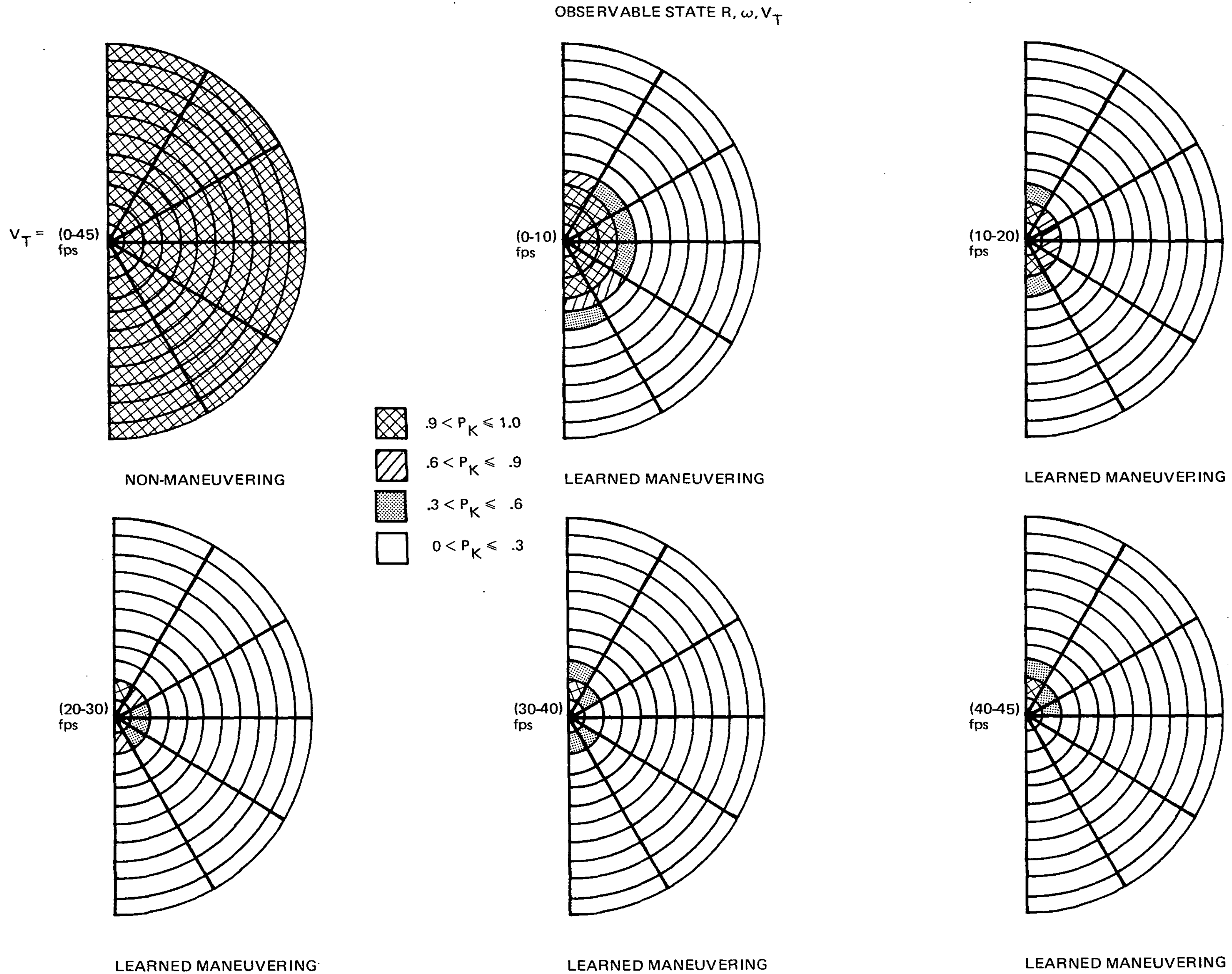


Fig. 25 Tank Survivability Results Study No. 2

operators of tank vehicles have commented that the baseline tank maximum deceleration capability of 1g employed in Studies 1 and 2 seemed high and that 0.5g was more reasonable in view of current operational capabilities on various types of terrain. The quantitative effect upon survivability when employing 0.5g maximum deceleration capability is shown in Fig. 26. The results show nearly identical missile kill probability levels for each of the tank initial speed ranges to those obtained in Study 2. (In fact, the small differences in results are within the range of statistical variability normally encountered in applications of the stochastic learning methodology.) The actual miss distances, however, were substantially reduced.

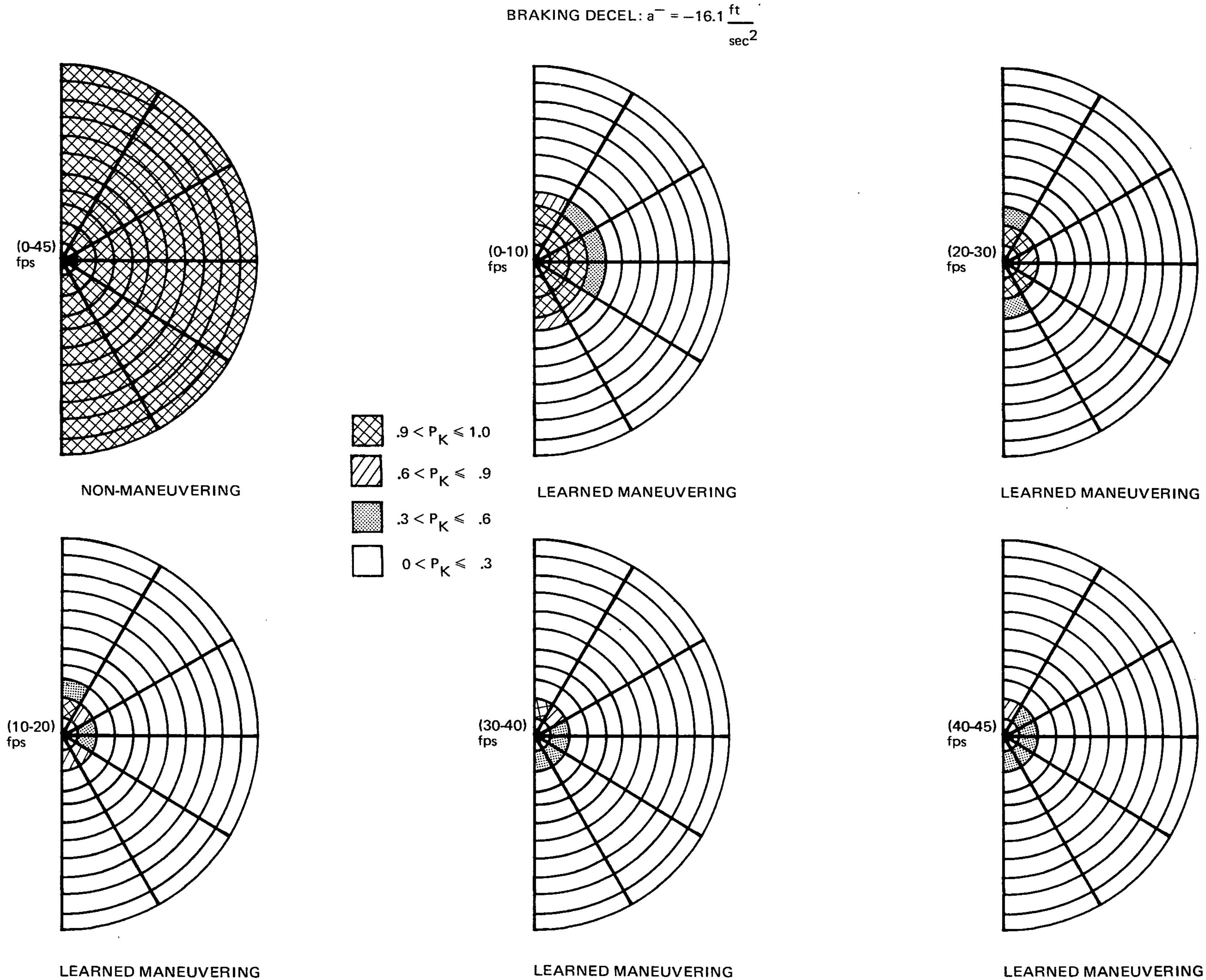


Fig. 26 Tank Survivability Results Study No. 3

6. CONCLUSIONS

A new computational tool has been developed for the quantification of tank vehicle survivability against ATGM threats. This computer program employs a stochastic learning method to determine the tank evasive maneuvers which maximize survival probability from all missile launch conditions. The optimized maneuver strategy is intimately related to the tank "information state" (which includes the threat warning system observables) and more specifically, is constructed as a discretized feedback control over the information state. The computational method is comprised of two phases; a learning phase within which the optimal evasion strategy for the tank is computed, and a statistics phase where the tank survivability is calculated (employing the optimal evasive strategy) for the entire space of missile initial conditions.

Three parametric cases were solved using representative tank/ATGM data. The first case established the baseline survivability results and associated evasive strategies against which the other cases can be compared. The assumed information state in this case comprised the threat warning thresholds of missile relative range and missile angle-off. The tank initial speed was 30 fps at the time of missile launch. The elements of the optimal evasive strategy as determined by the solution were, 1) maximal performance turns to achieve beam aspect with the oncoming missile followed by 2) a braking deceleration maneuver initiated at the 250m relative range warning threshold. (This computed strategy has been empirically verified in recent field trials with live firings of similar missiles using inert warheads against moving tank vehicles.) This result illustrated the joint importance of the tank threat warning cues and braking deceleration performance in achieving high survivability.

In the second case the tank information state was augmented to include tank speed thresholds in addition to the threat warning thresholds of the first case. The resulting strategy for this case again emphasized the carefully timed braking deceleration maneuver and maximal performance turns as before for high initial tank speeds. However, a maximal forward acceleration performance maneuver was found critical to achieving high survivability for close-in missile launches at low tank speeds. In these cases the acceleration maneuver was necessary to achieve a higher tank speed to more effectively employ the final deceleration maneuver.

For the last case, the tank braking deceleration performance was reduced from 1 to 0.5g in response to comments from field officers that the 1g performance was an overestimate of current system capability and that the 0.5g value more closely approximated their experience with these vehicles on various types of terrain. The computed survivability results and associated maneuver strategy for the 0.5g case were virtually identical to those computed for the 1g case. Of course, this sensitivity result is specific to the threat missile system considered and general conclusions would require more extensive studies.

The computer program is relatively inexpensive to use from a running time standpoint, requiring approximately 1/2 hour on an IBM 370/168 computer for both learning and statistics phases. This program can be a valuable asset in a wide spectrum of future applications dealing with tank survivability and tactics development against ATGM threats. Four of the more important applications are:

1) Vehicle preliminary design, involving evaluation of overland speed/maneuver performance requirements to achieve required survivability levels;

2) Threat warning system preliminary design, involving the systematic evaluation of the observables and their thresholding to meet required survivability levels. This would strongly impact future tank defensive computer requirements. (While this development has focused on a single ATGM threat, multiple threat cases involving both staggered and simultaneous launches can be similarly addressed);

3) Armor preliminary design, including the effects on survivability of armor distribution, interior compartmenting and exterior envelope design. Tradeoffs for various armor/maneuver agility/threat warning conceptualizations can be assessed.

4) Countermeasure requirements, including smoke and flare deployment strategies and their effect upon survivability. The elemental maneuver control set in the current program can easily be augmented to include countermeasure control choices.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 12285	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Development and Analysis of Tank Evasion Strategies in Missile Effectiveness Models		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER RE-544
7. AUTHOR(s) M. Falco G. Carpenter		8. CONTRACT OR GRANT NUMBER(s) DAAE07-76-C-0106
9. PERFORMING ORGANIZATION NAME AND ADDRESS Grumman Aerospace Corporation Bethpage, New York 11714		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Tank-Automotive Research and Development Command Warren, Michigan 48090		12. REPORT DATE June 1977
		13. NUMBER OF PAGES 53
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Tank Survivability Analysis, Anti-Tank Missile Effectiveness, Evasive Strategy Development, Threat Warning System, Math Models, Computer Analysis, Optimization

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a new methodology with which to analyze the survivability of tank vehicles to anti-tank missile threats. The approach employs elements of optimal control theory, stochastic learning theory, and dynamical simulation in a computational method which determines tank evasive maneuvering strategy as an integral part of the survivability analysis. The method develops an optimal strategy in the sense of maximizing tank survival probability for all missile launch conditions. The strategy is in the form of a feedback control policy based upon a discretized set of information states which are assumed available to the tank commander as visual or warning system cues. Computational results for both the survivability and associated optimal evasive maneuvering are presented for an M-60 class tank vehicle and an anti-tank missile representative of an upgraded foreign threat system. The results illustrate how the methodology can be employed to assist in quantifying survivability tradeoffs involving tank threat warning systems, evasive maneuver computer systems, and acceleration, deceleration, and turning performance specifications.